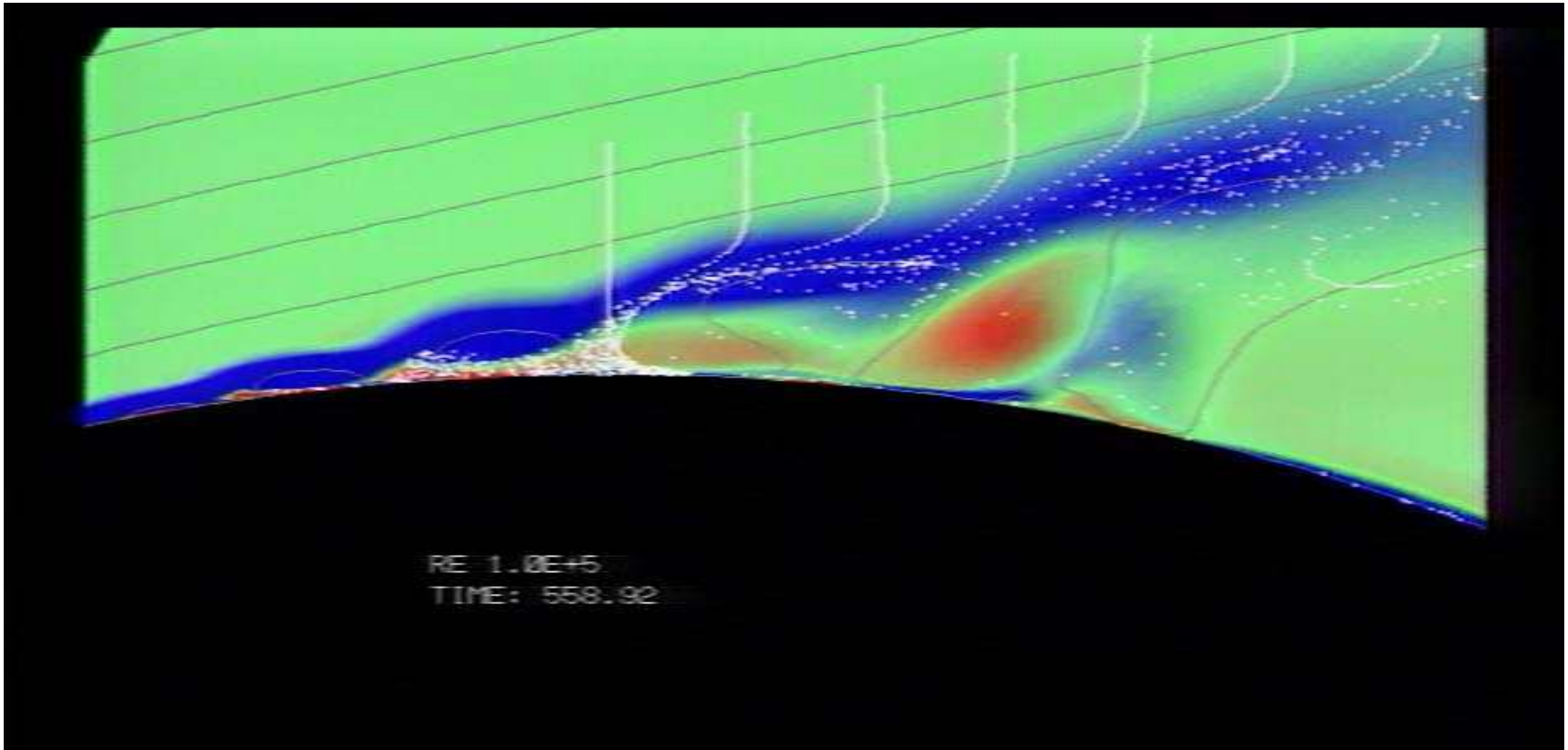
Boundary Layer Theory

Professor
Ugur GUVEN
Aerospace Engineer
Spacecraft Propulsion Specialist

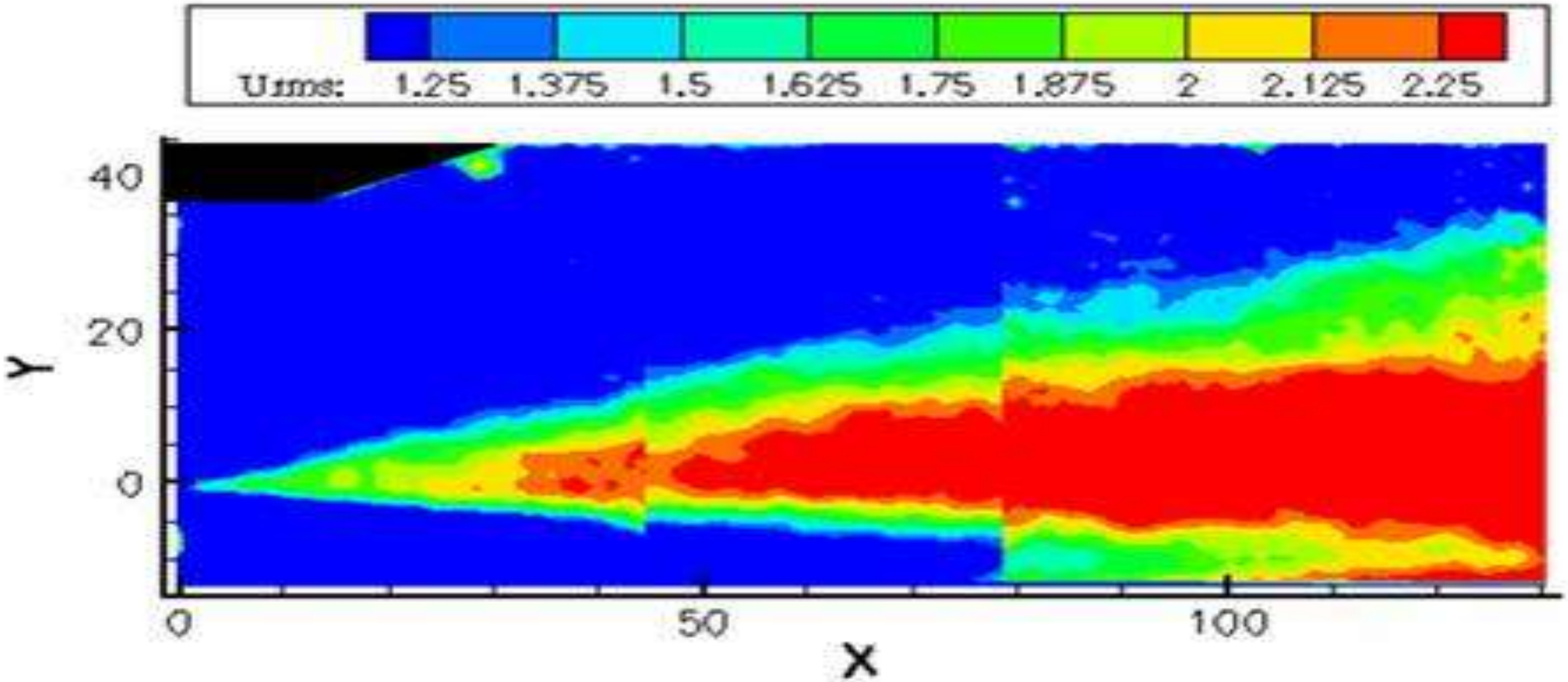
Boundary Layer Definition

- ***<u>Boundary Layer</u>*** is the thin boundary region between the flow and the solid surface, where the flow is retarded due to friction between the solid body and the fluid flow.



Shear Layer

- ***<u>Shear Layer</u>*** is the thin boundary region between the two flows with different velocities.



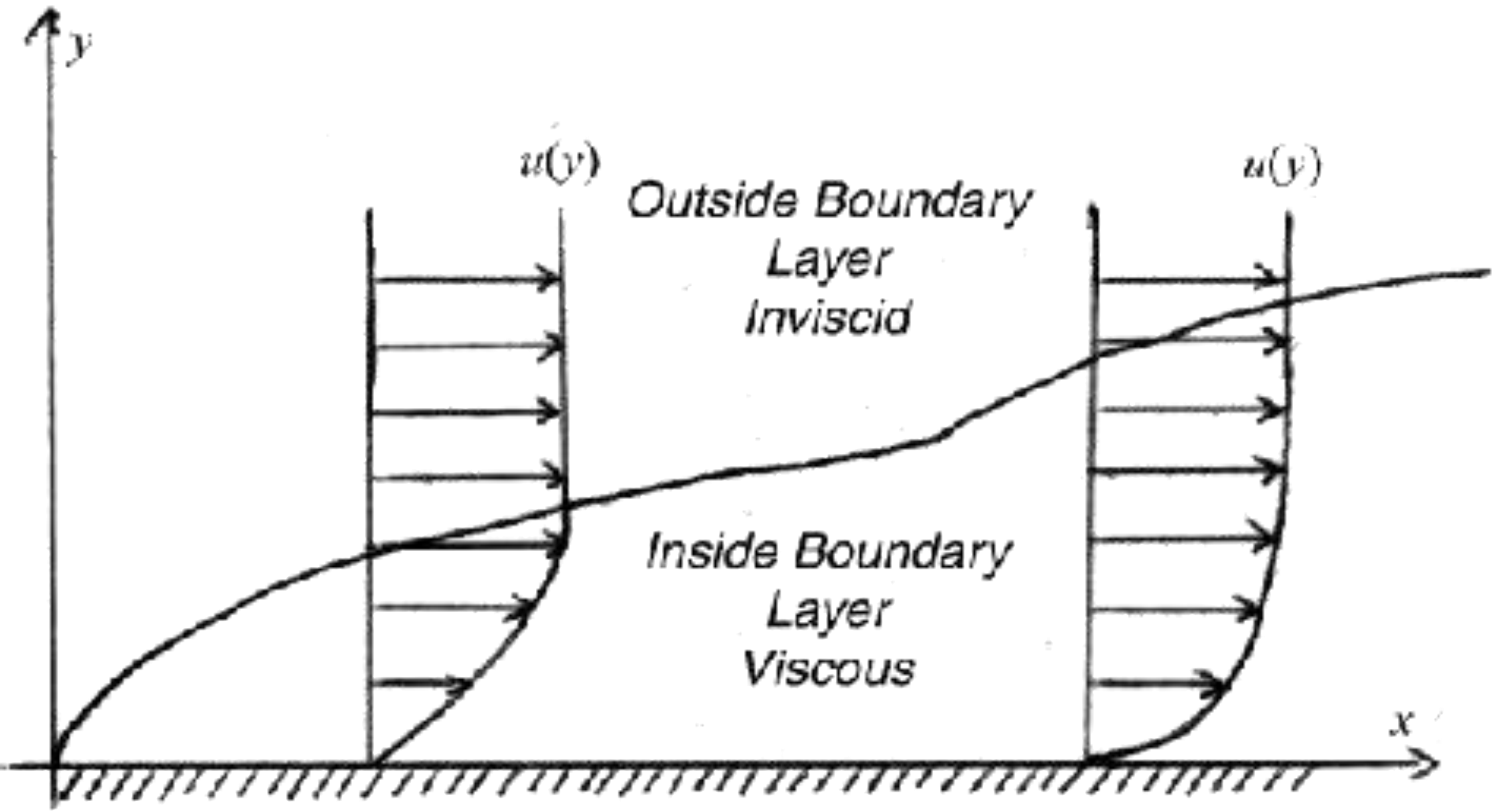
What is the Significance of Boundary Layers?

- Although friction exists in all types of flow, practically it is only of consequence in the thin region separating the flow and the solid body.
- Hence, as far as the physical system is concerned, the boundary layer is the region where mass transfer, momentum transfer, heat transfer, friction effects and all viscosity effects are felt.

How Does a Boundary Layer Help Engineers!

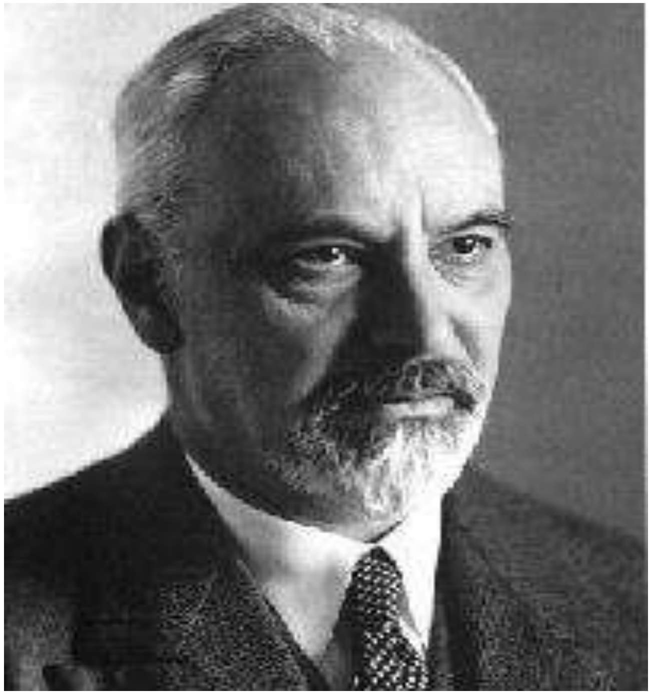
- This means that instead of solving for the whole Navier Stokes equation set for the full flow, we can approximate a solution by ***solving for the boundary layer where the viscous effects are felt***.
- Thus, in order to calculate skin friction and aerodynamic heating at the surface, you only have to account for friction and thermal conduction within the thin boundary layer. Hence; you wont need to analyze the large flow outside the boundary layer

Calculation of a Flow with a Boundary Layer

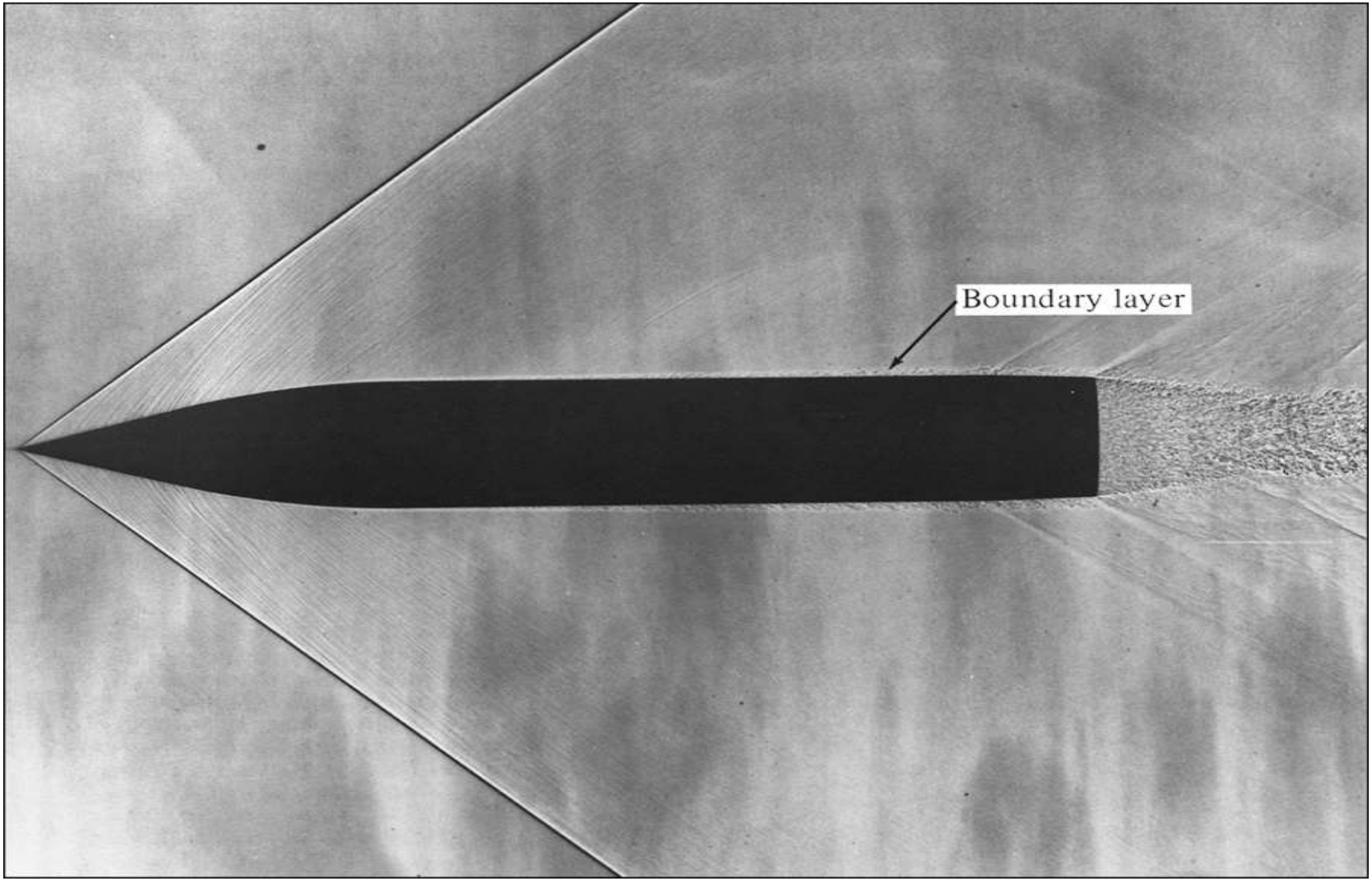


Boundary Layer Equations

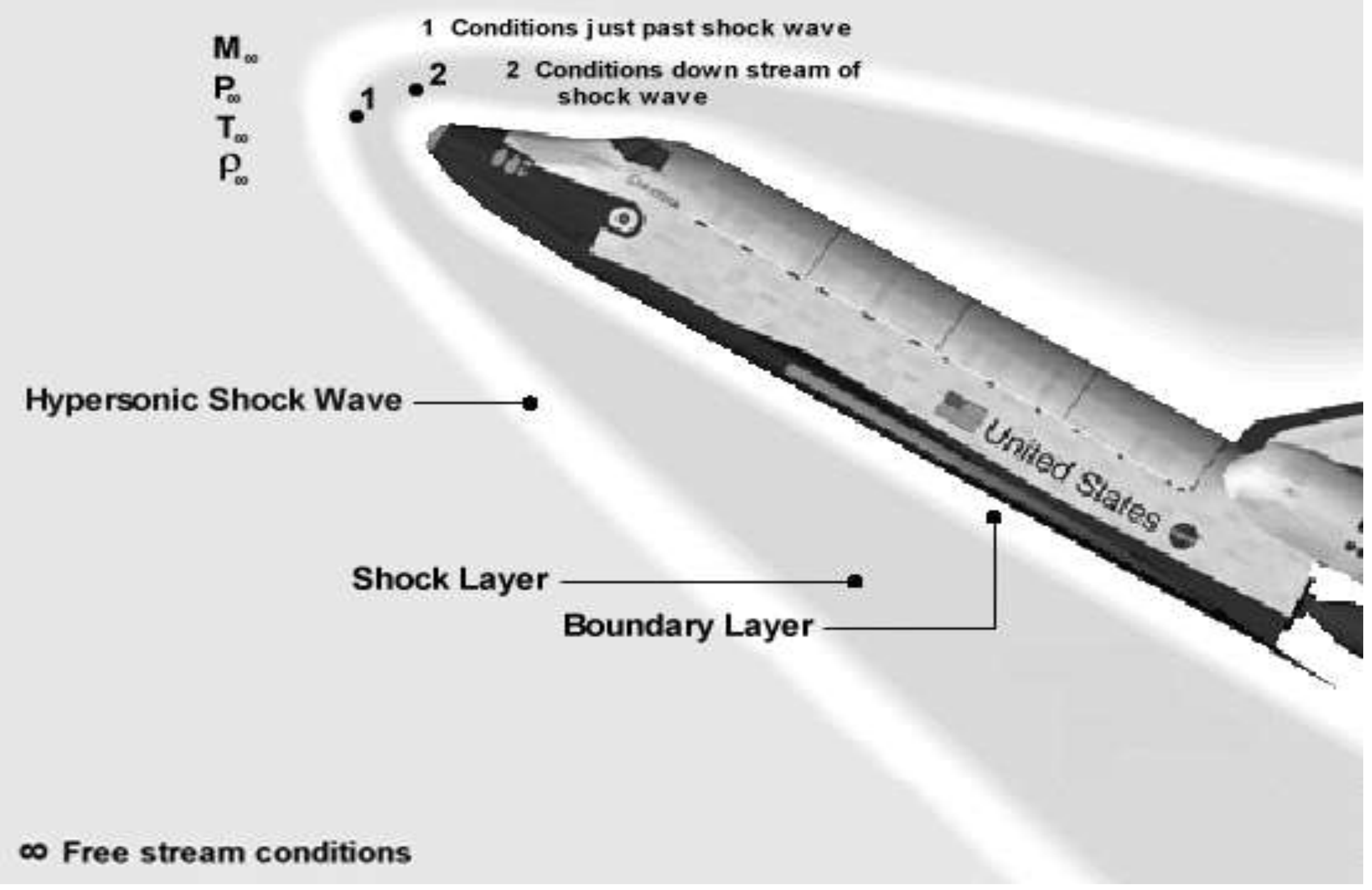
- Boundary Layer concept was founded by Ludwig Prandtl and it revolutionized the concept of solving Navier Stokes questions.
- Boundary Layer Equations are partial differential equations that apply inside the boundary layer.



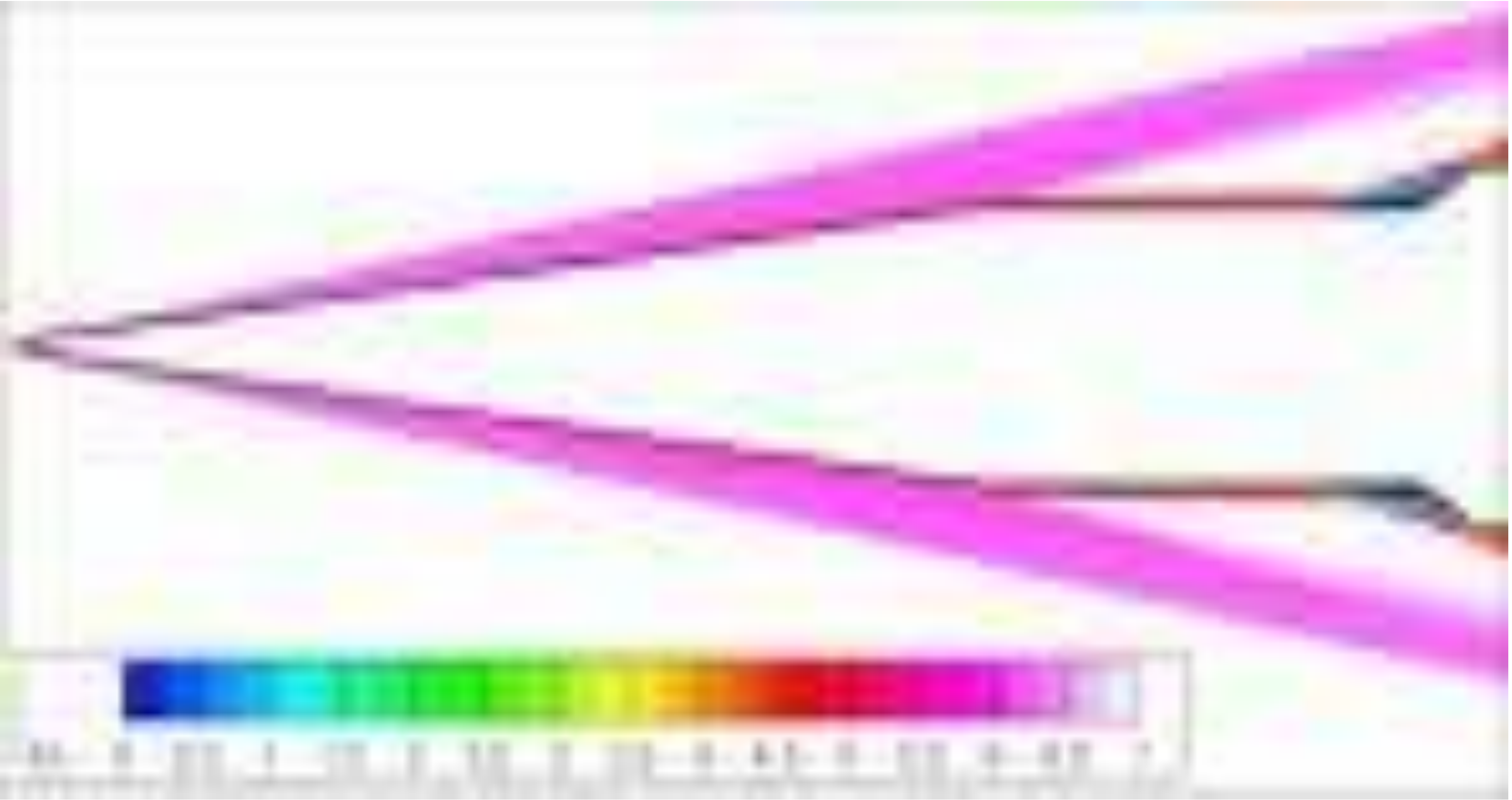
Representation of a Boundary Layer



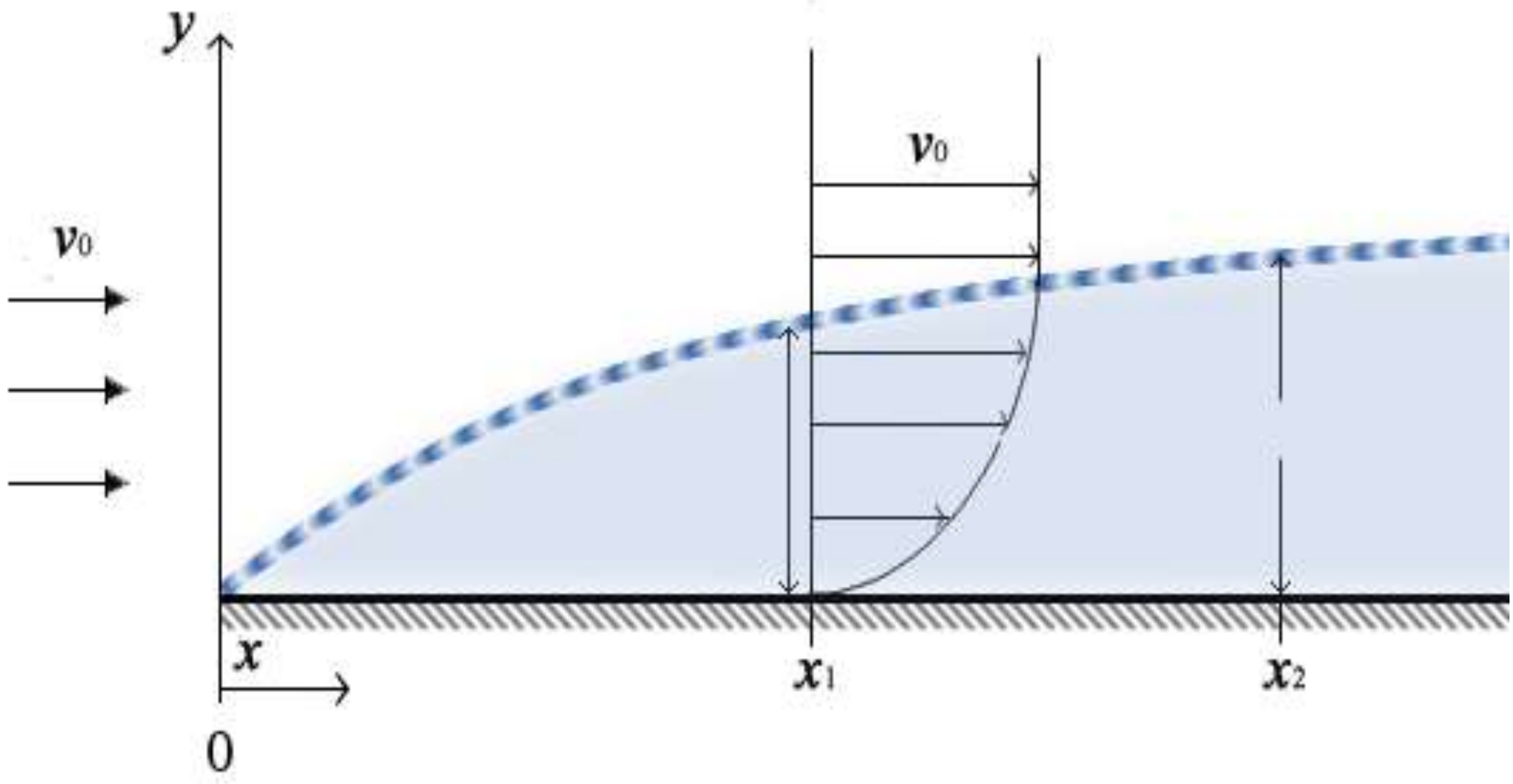
Representation of a Boundary Layer



Representation of a Boundary Layer

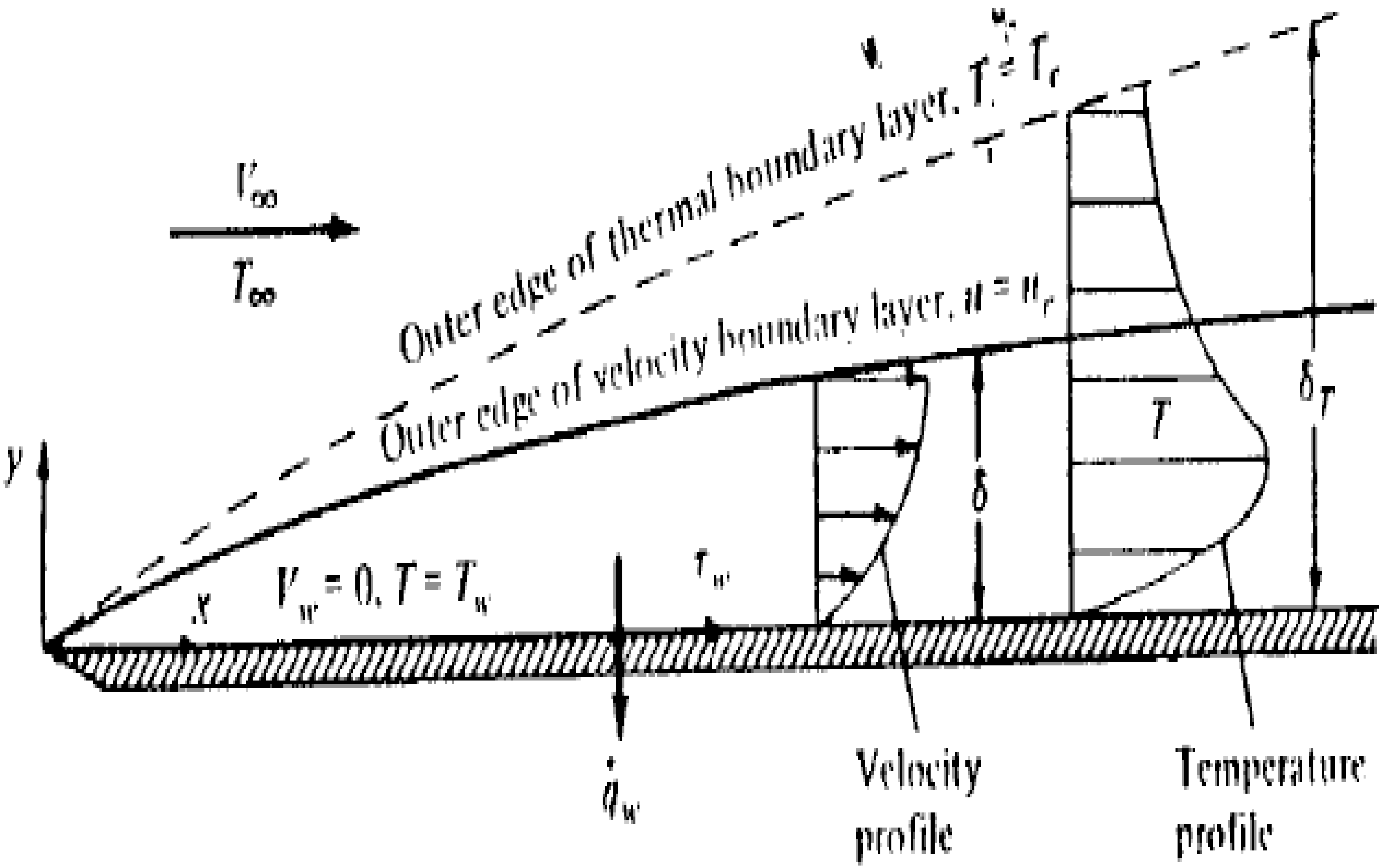


Shape of a Boundary Layer

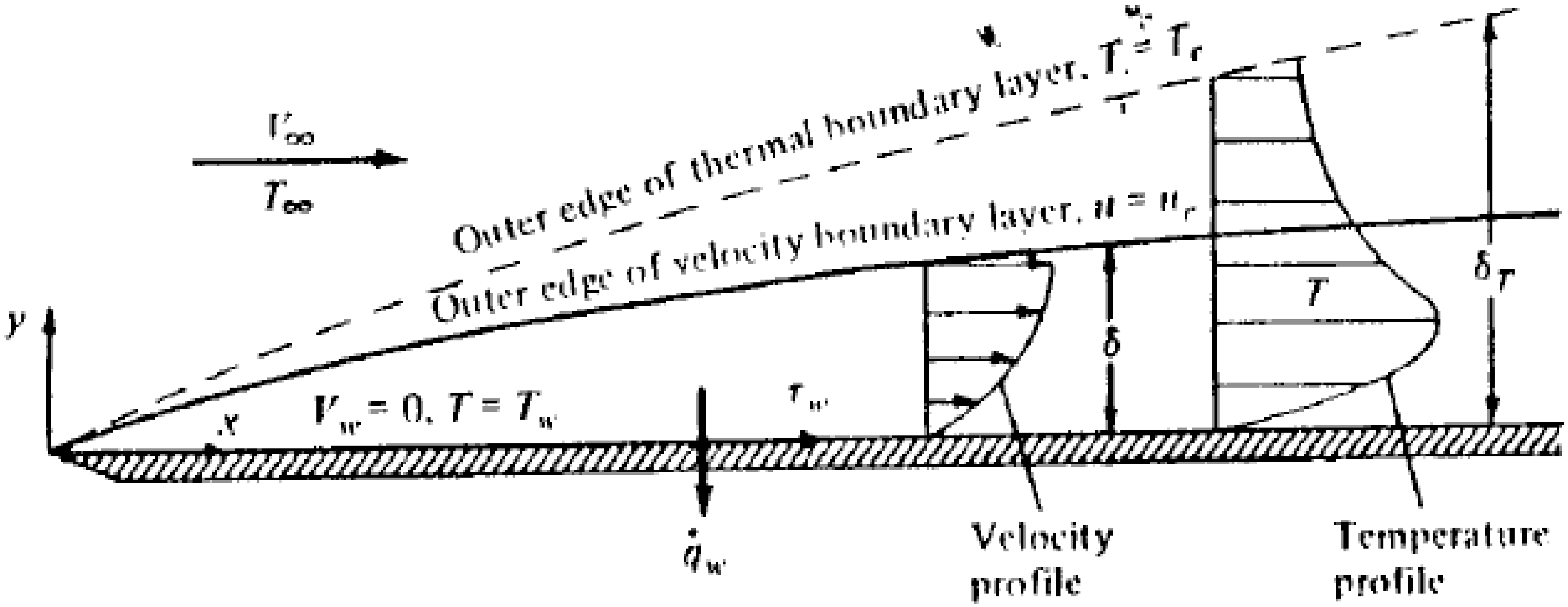


Because of No-Slip condition, the velocity of the fluid is Zero at the surface and it gradually increases.

Boundary Layer Properties



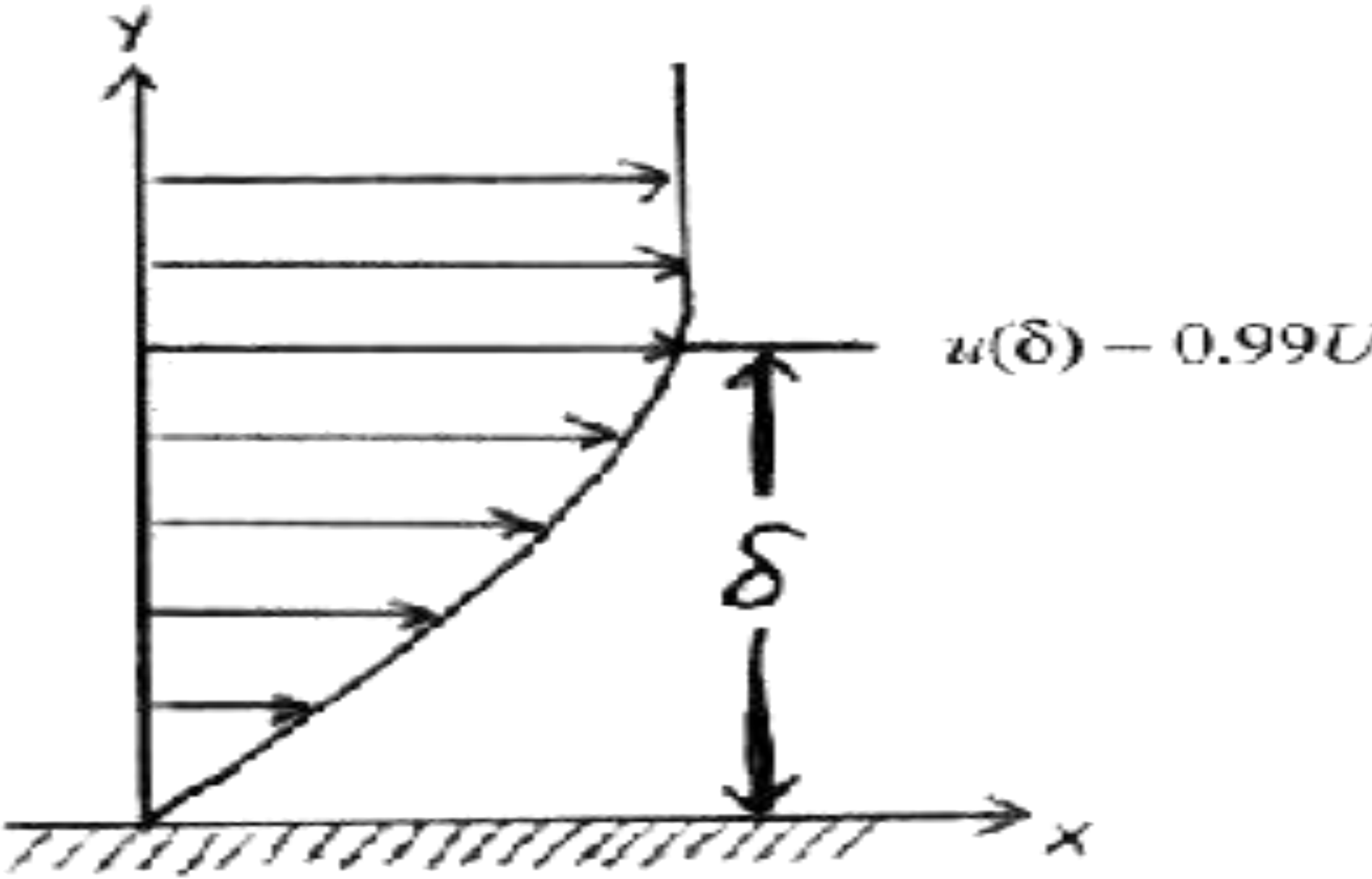
Boundary Layer Properties



δ is defined as that distance above the wall where $u = 0.99u_e$
u_e is the velocity at the outer edge of the boundary layer.

$T = T_w$ at $y = 0$ to $T = 0.99T_e$ at $y = \delta_T$

Velocity Boundary Layer



Two Boundary Layers

- Hence, two boundary layers can be defined:

  1) Velocity Boundary Layer with Thickness δ

  2) Temperature Boundary Layer with Thickness δ_T

$$\text{if } Pr = 1, \text{ then } \delta = \delta_T$$

$$\text{if } Pr > 1, \text{ then } \delta_T < \delta$$

$$\text{if } Pr < 1, \text{ then } \delta_T > \delta$$

For air at standard conditions

$Pr = 0.71$; hence, the thermal boundary layer is thicker than the velocity boundary

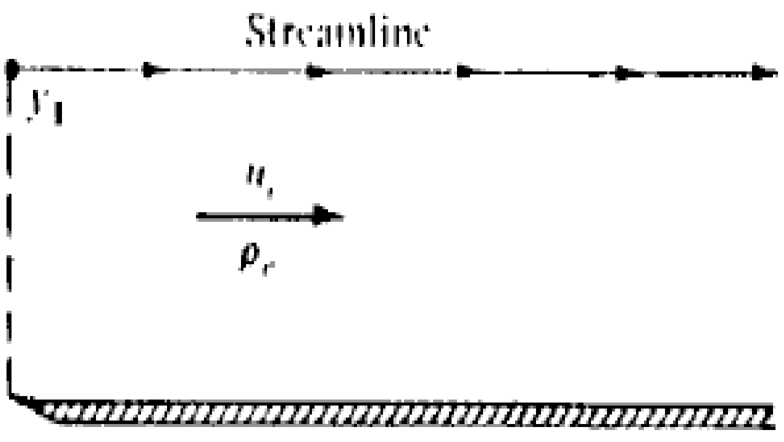
Properties of the Boundary Layer

- Hence since Prandtl number is 0.71 for air at standard conditions, the thermal boundary layer is thicker then the velocity boundary Layer. (Remember Prandtl number is a function of temperature).
- Both boundary layer thicknesses increase with distance from the leading edge.

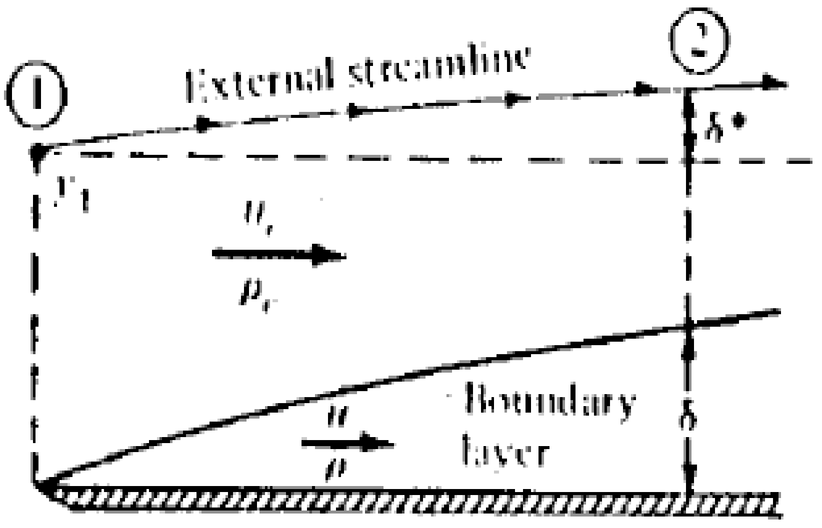
$$\tau_w = \tau_w(x) \text{ and } \dot{q}_w = \dot{q}_w(x).$$

Displacement Thickness

- ***Displacement Thickness*** is the distance between the actual body surface and the effective body.
- Displacement Thickness can also be visualized as the index proportional to the missing mass flow due to the presence of the boundary layer.



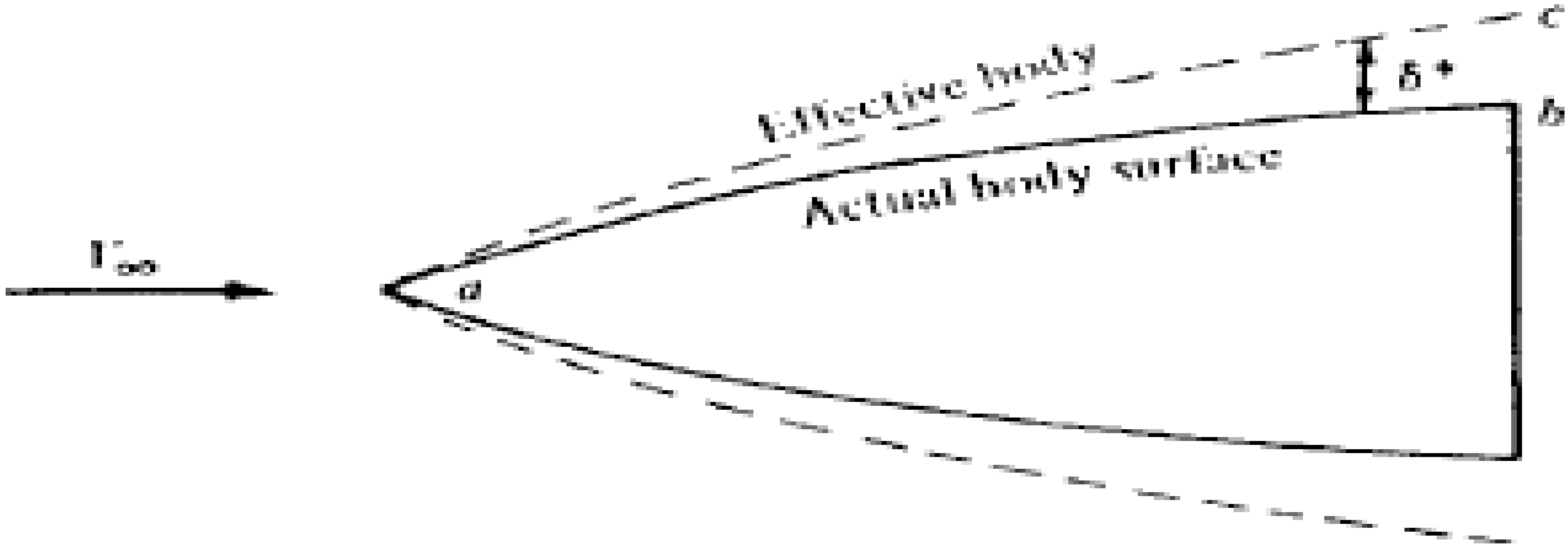
Hypothetical flow with no boundary layer (inviscid case)



Actual flow with a boundary layer

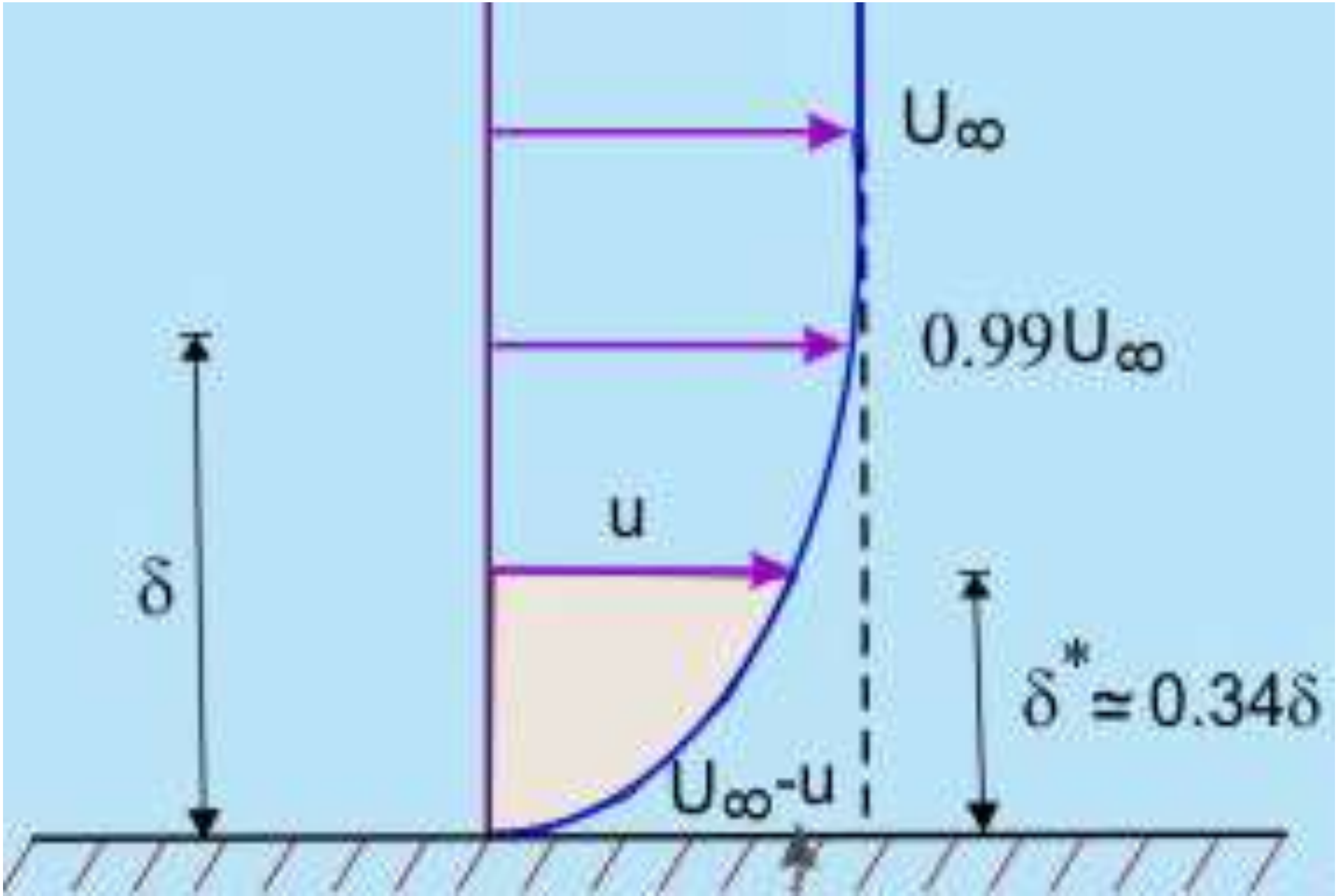
Displacement Thickness

$$\delta^* \equiv \int_0^{y_1} \left(1 - \frac{\rho u}{\rho_e u_e} \right) dy \qquad \delta \le y_1 \to \infty$$



δ^* is usually smaller than δ; typically, $\delta \approx 0.3\delta$.

Displacement Thickness

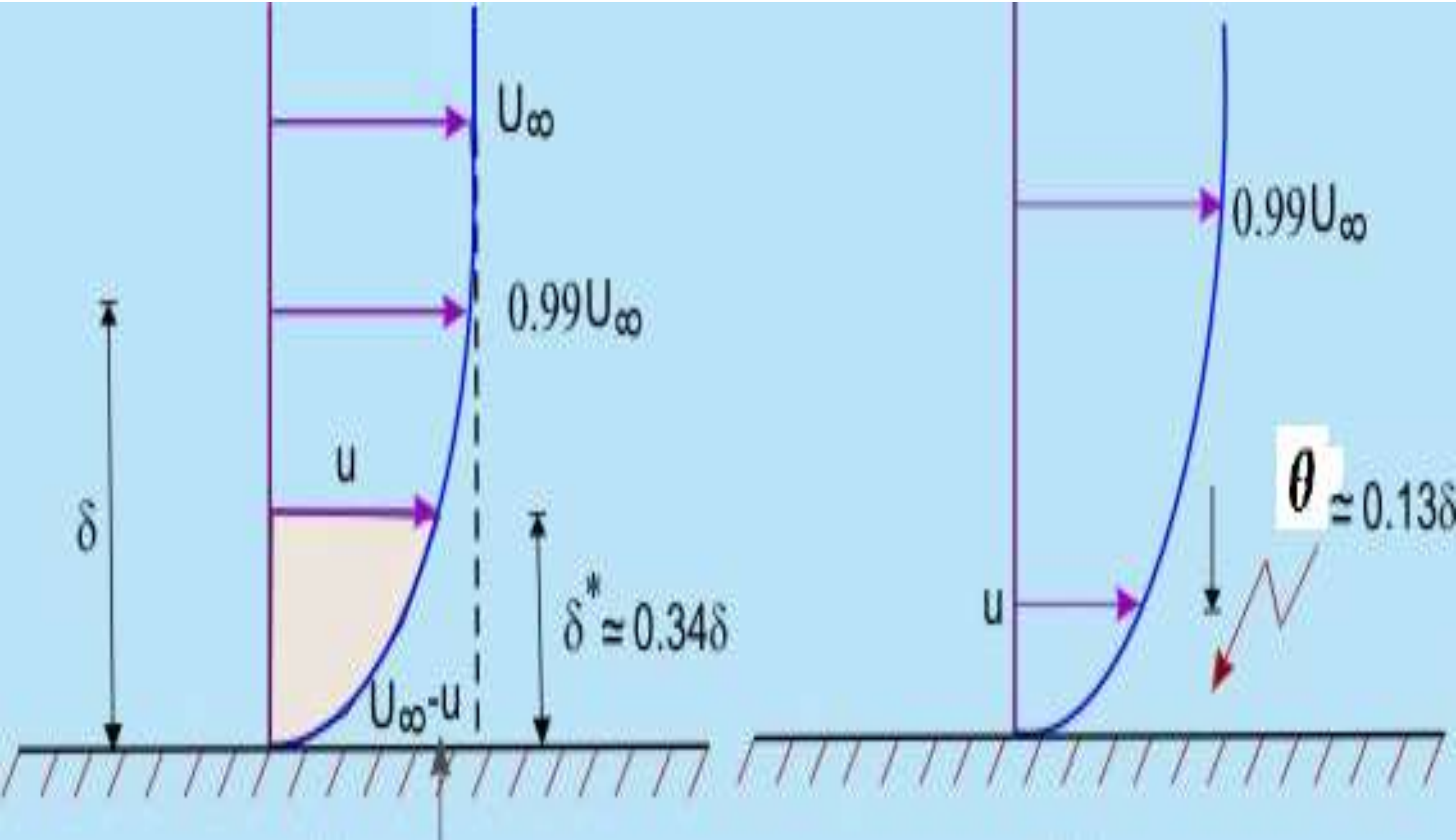


Momentum Thickness

- **ϴ or Momentum Thickness** is an index that is proportional to the decrement in momentum flow due to the presence of the boundary layer.
- **ϴ** is also the height of a hypothetical stream, which is carrying the missing momentum flow at free stream conditions.

$$\theta \equiv \int_0^{y_1} \frac{\rho u}{\rho_e u_e}\left(1 - \frac{u}{u_e}\right) dy \qquad \delta \leq y_1 \rightarrow \infty$$

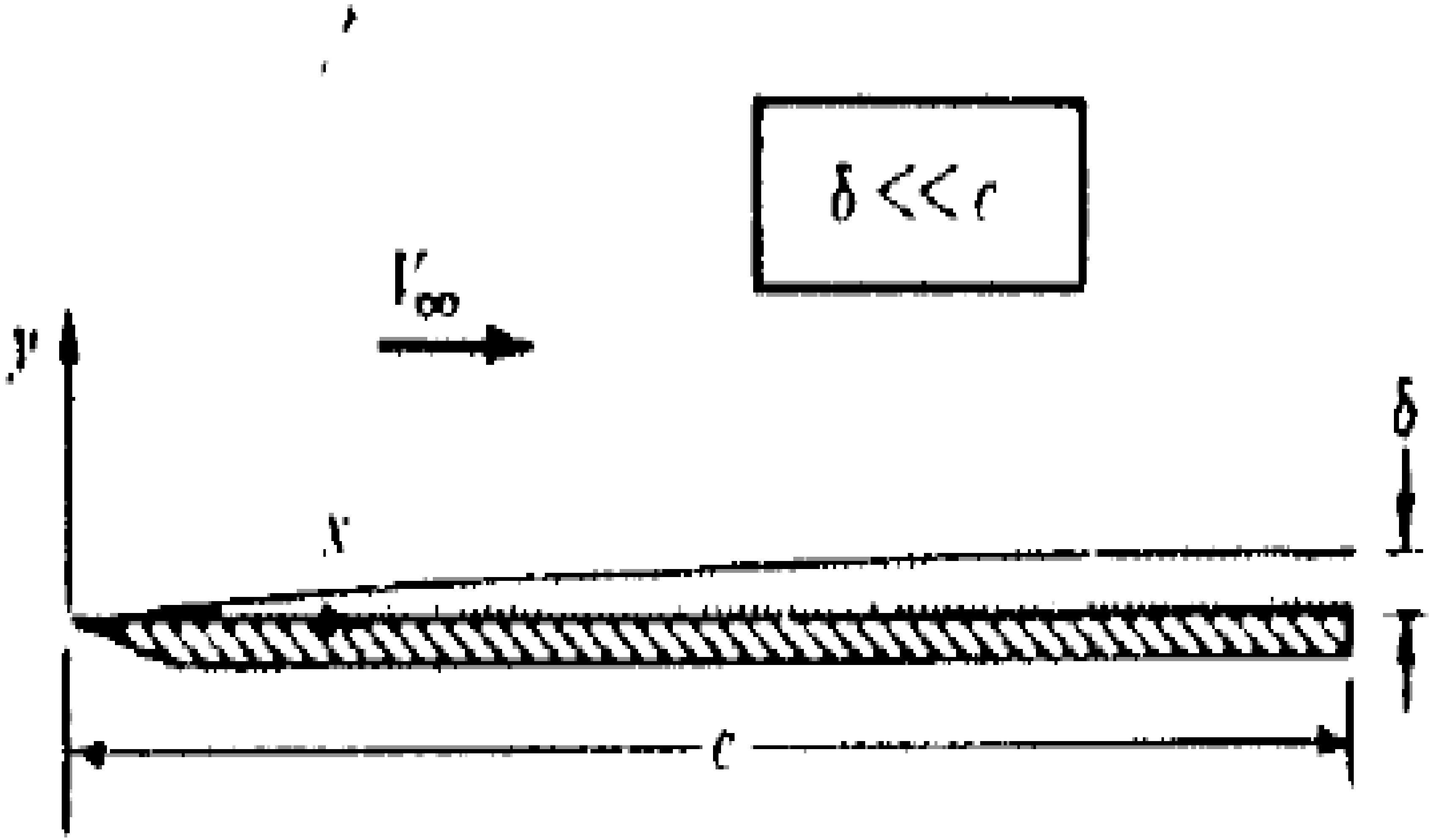
Displacement and Momentum Thickness



Generality of Boundary Layers

- The concepts outlined above such as velocity boundary layer, thermal boundary layer, displacement thickness and momentum thickness hold true for all types of boundary layers in viscous flow. (Compressible/Incompressible, Laminar/Turbulent Flow)
- In order to solve for a boundary layer, the boundary layer must be very thin in comparison $\delta \ll c$

Solution of Boundary Layer



Boundary Layer Equations

Continuity: $$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

x momentum: $$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

y momentum: $$\frac{\partial p}{\partial y} = 0$$

Energy: $$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{dp_e}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2$$

Boundary Conditions

At the wall: $y = 0 \quad u = 0 \quad v = 0 \quad T = T_w$

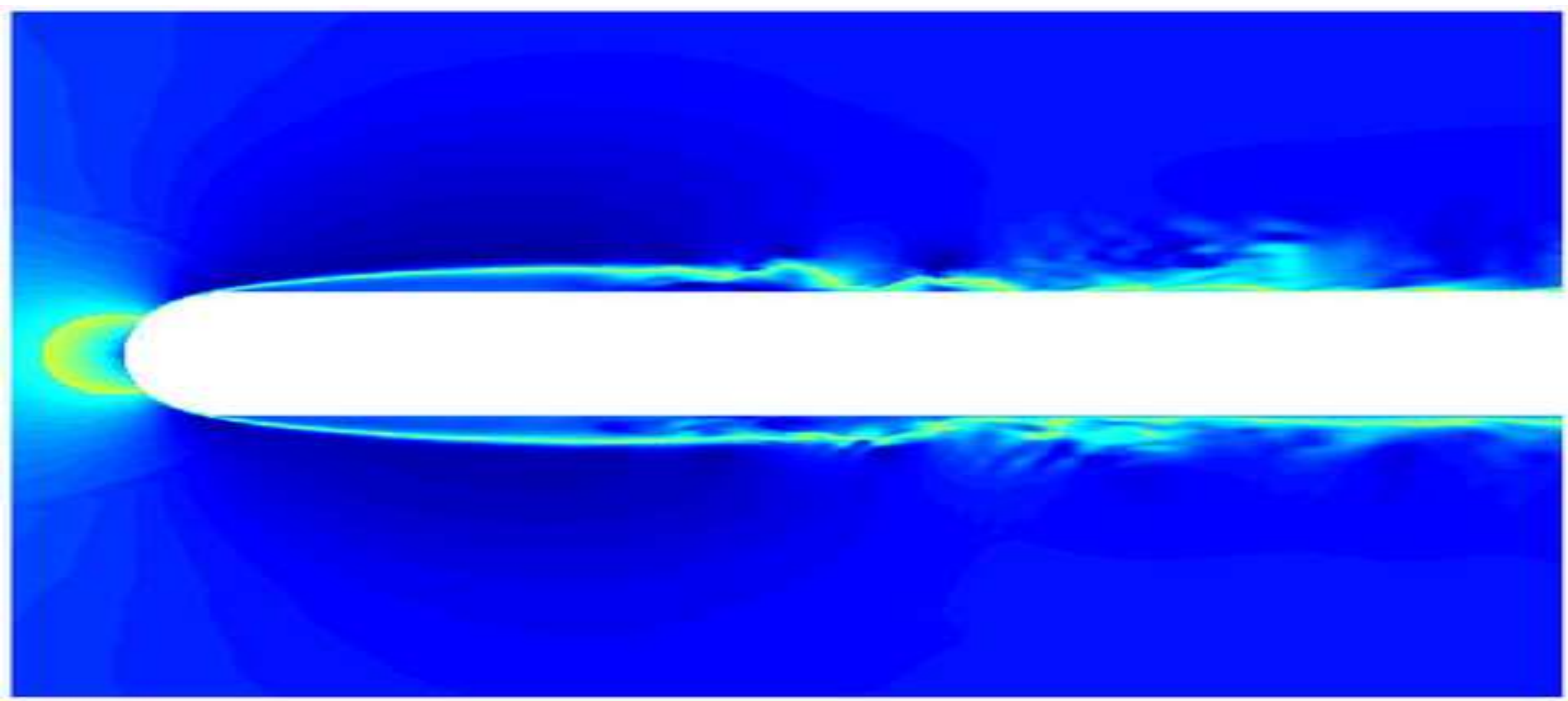
At the boundary layer edge: $y \to \infty \quad u \to u_e \quad T \to T_e$

$$p = \rho R T$$

$$h = c_p T$$

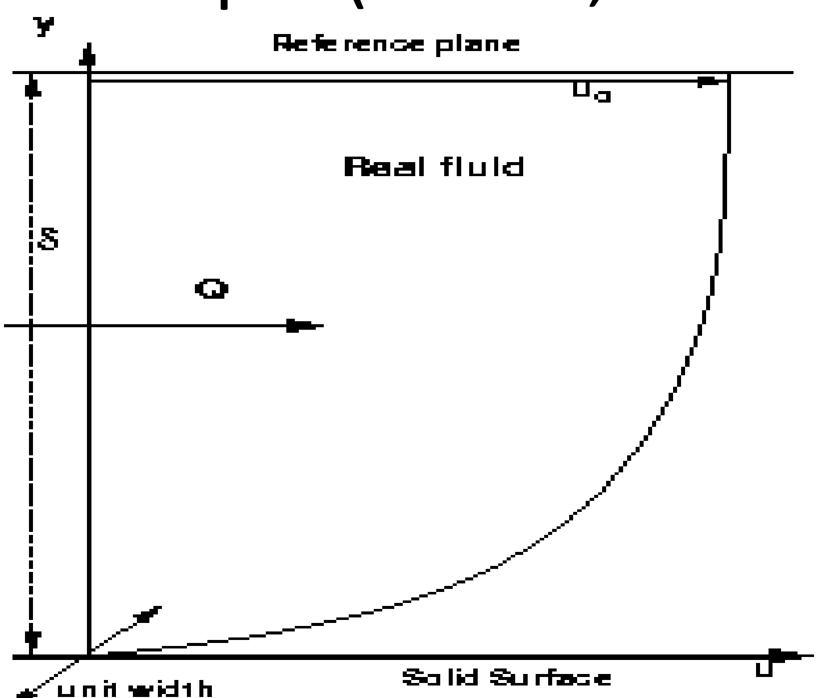
Solution of Boundary Layer Problems

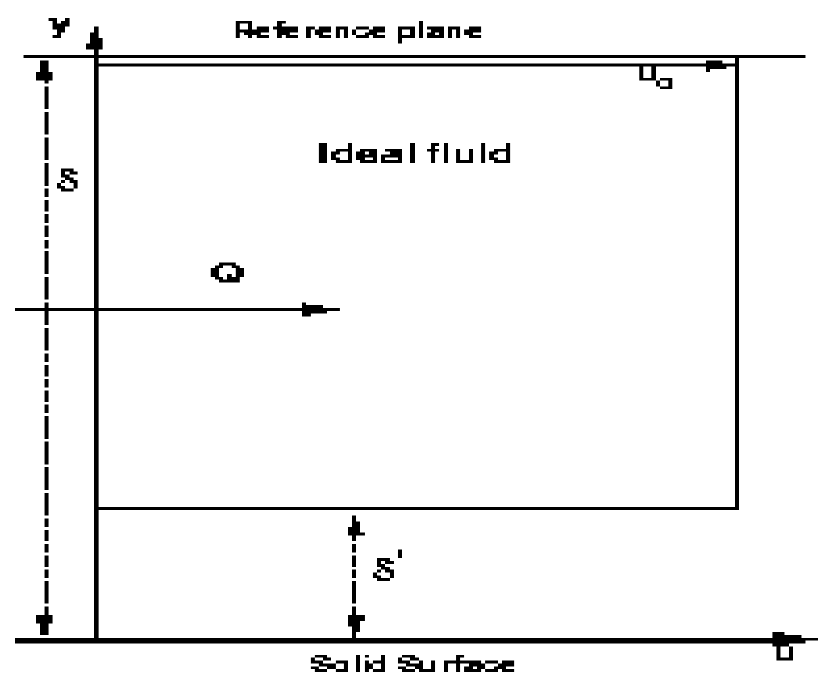
- Solve Boundary Layer Equations
- Using Velocity and Temperature values from above equations solve for shear stress and for heat transfer using the Fourier formula.
- Use Numerical Methods to iterate



Properties of Boundary Layers

- The external flow reacts to the edge of the boundary layer just as it would to the physical surface of an object. So the boundary layer gives any object an "effective" shape which is usually slightly different from the physical shape. (Hence, the displacement thickness)

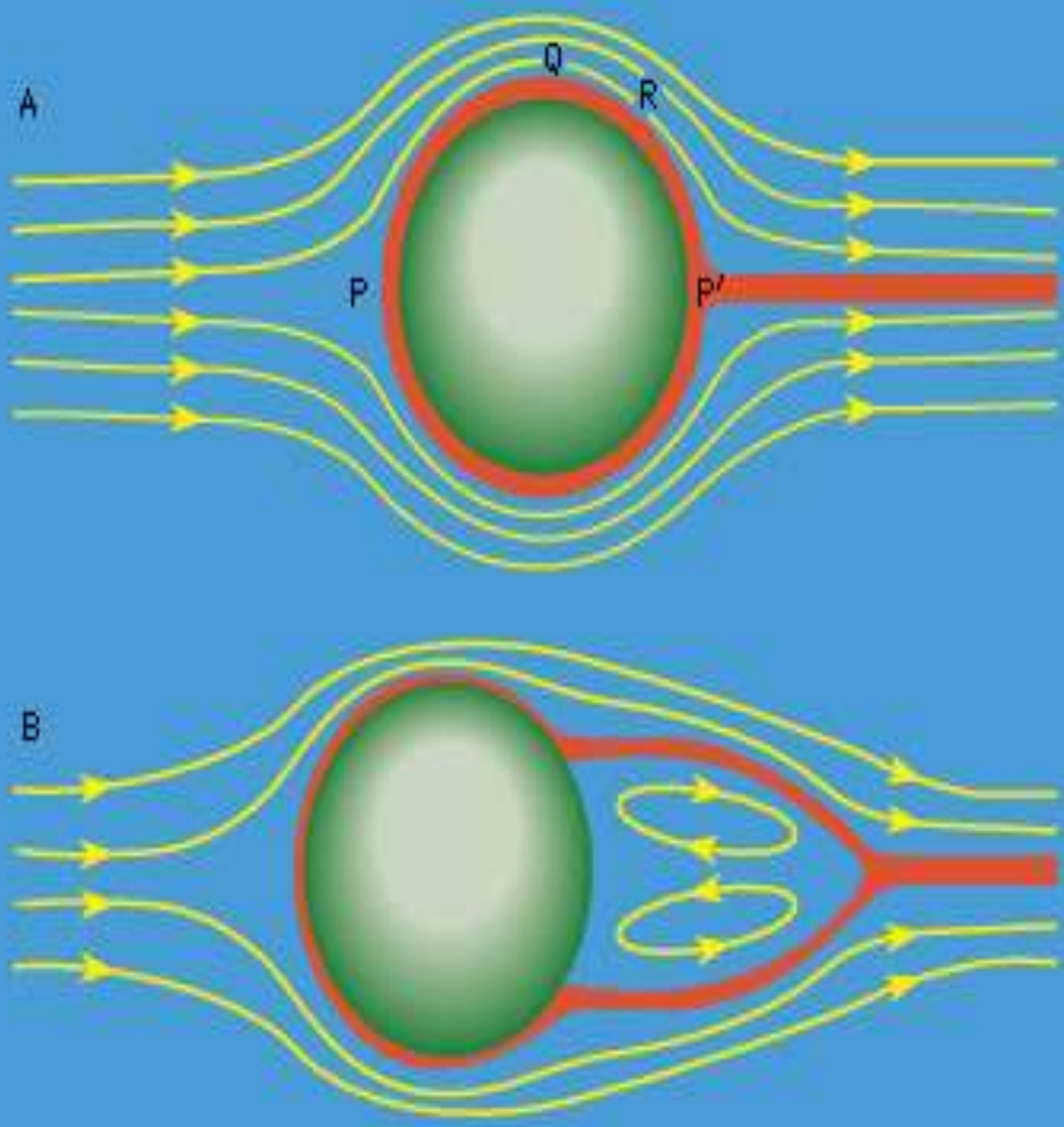




Separation of Boundary Layers

- ***The boundary layer may lift off or "separate" from the body and create an effective shape much different from the physical shape***. This happens because the flow in the boundary has very low energy (relative to the free stream) and is more easily driven by changes in pressure.
- **Boundary layer separation** occurs when the portion of the boundary layer closest to the wall reverses in flow direction. As a result, the overall boundary layer initially thickens suddenly and is then forced off the surface by the reversed flow at its bottom
-

Separation of Boundary Layer

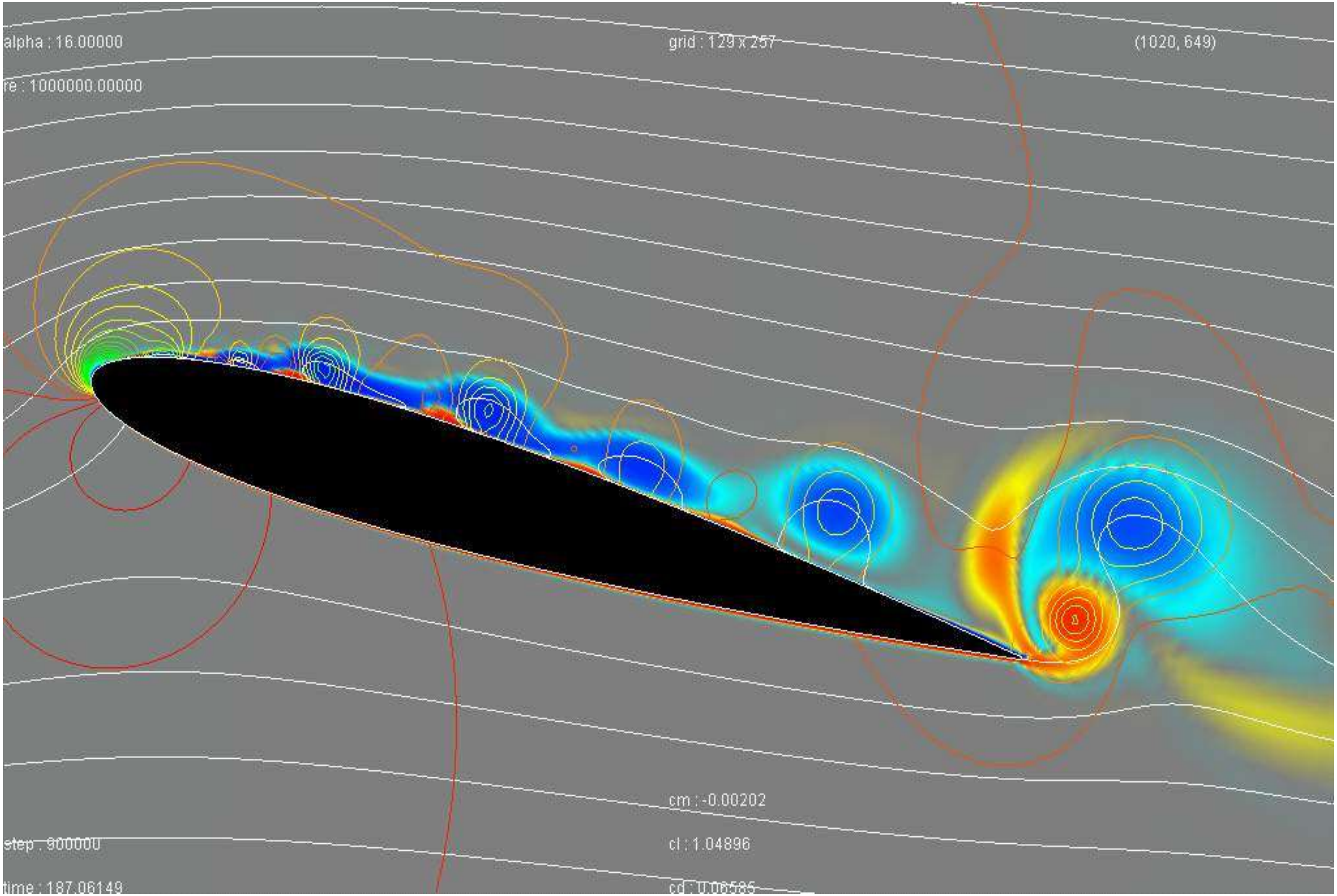


The fluid is free of vorticity outside a boundary layer, which is represented by shading. In (A) the boundary layer is still attached to the sphere, though it continues downstream of it. In (B) it has separated, and an eddy has formed behind the sphere.

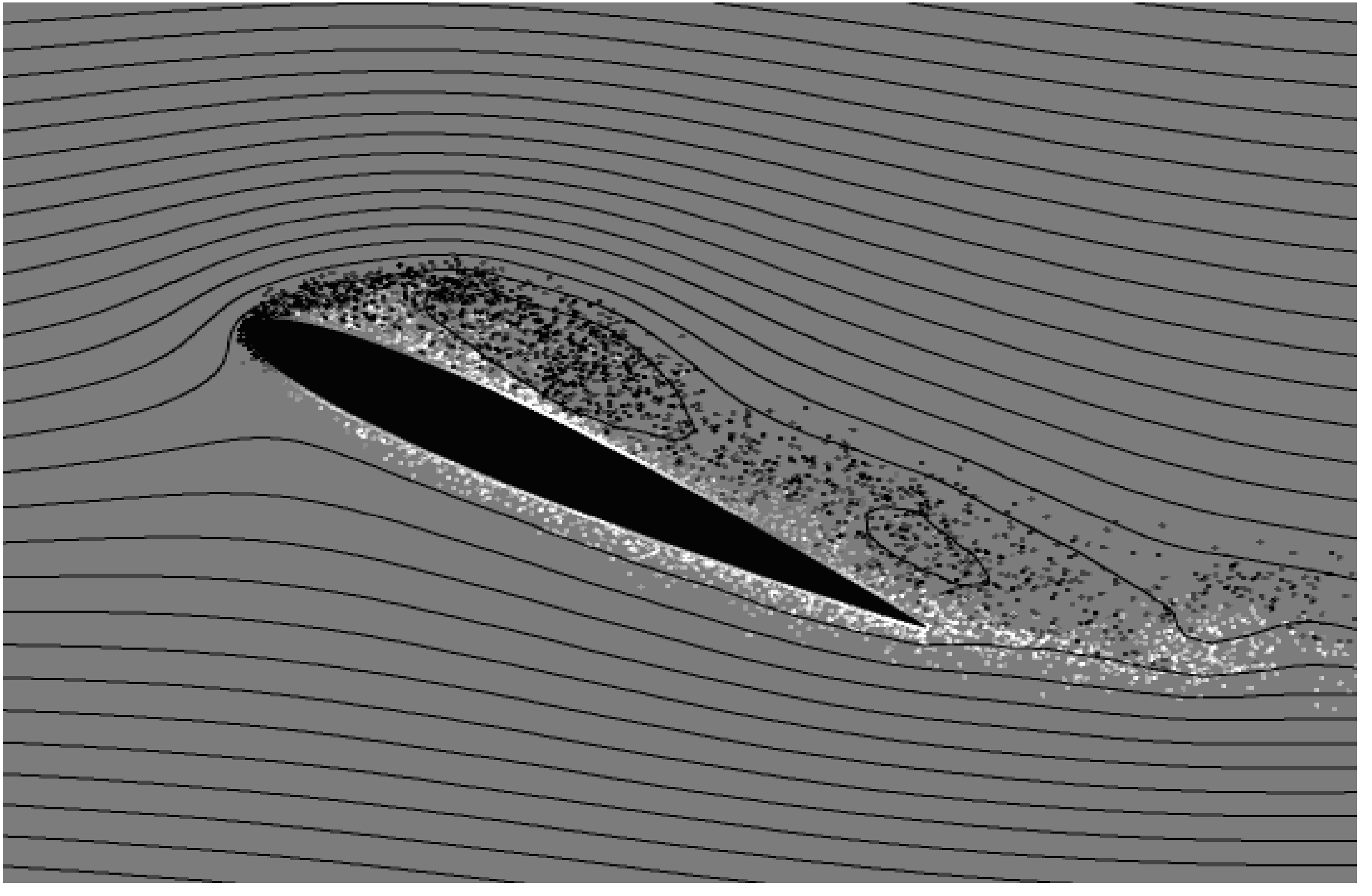
How Does Boundary Separation Occur?

- *Separation is bound to occur in a sufficiently large adverse pressure gradient.*
- Boundary layers tend to separate from a solid body when there is an increasing fluid pressure in the direction of the flow.
- Increasing the fluid pressure is akin to increasing the potential energy of the fluid, leading to a reduced kinetic energy and a deceleration of the fluid

Flow Separation in an NACA Airfoil

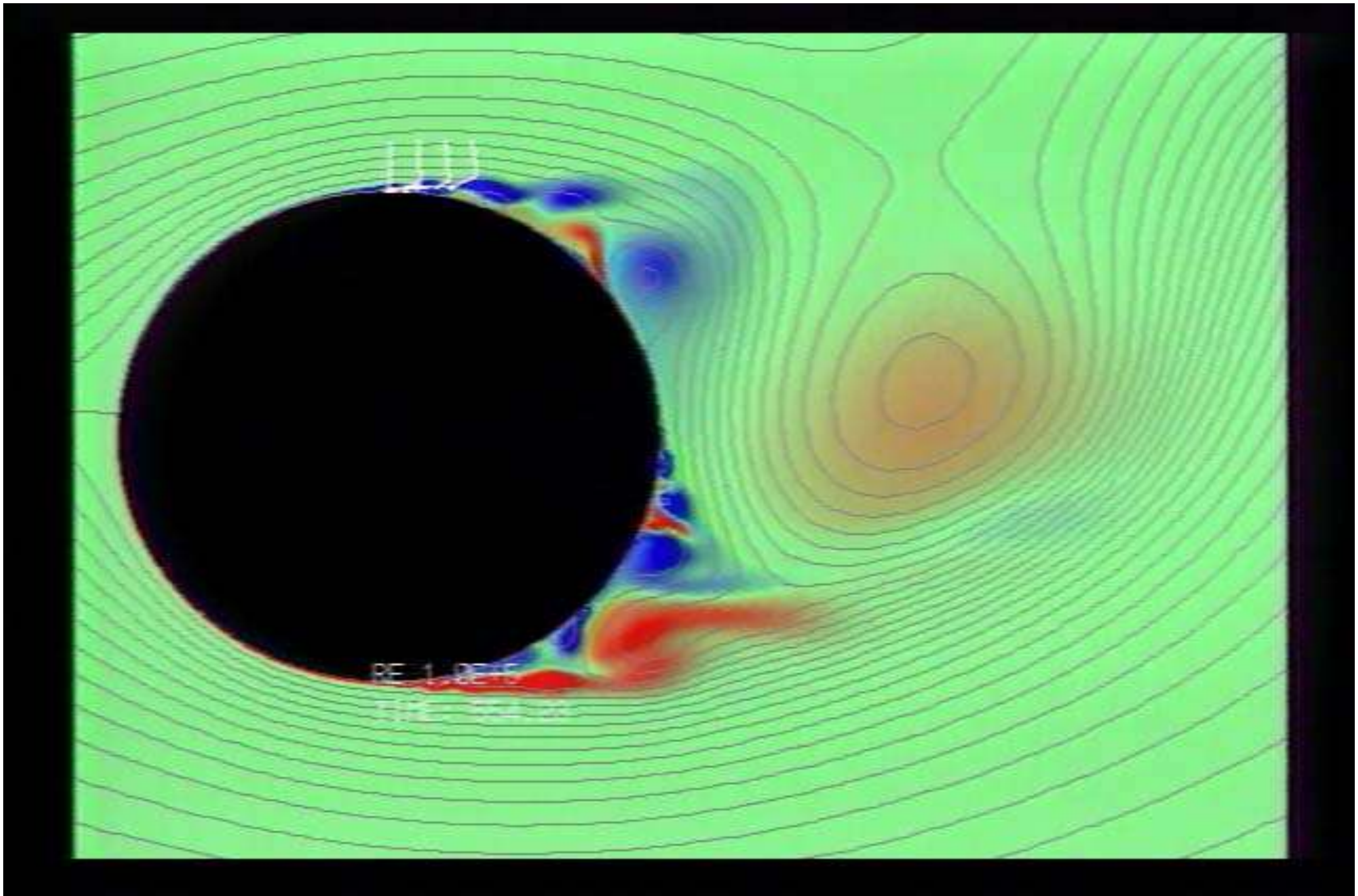


Flow Separation



Flow Separation on a Sphere

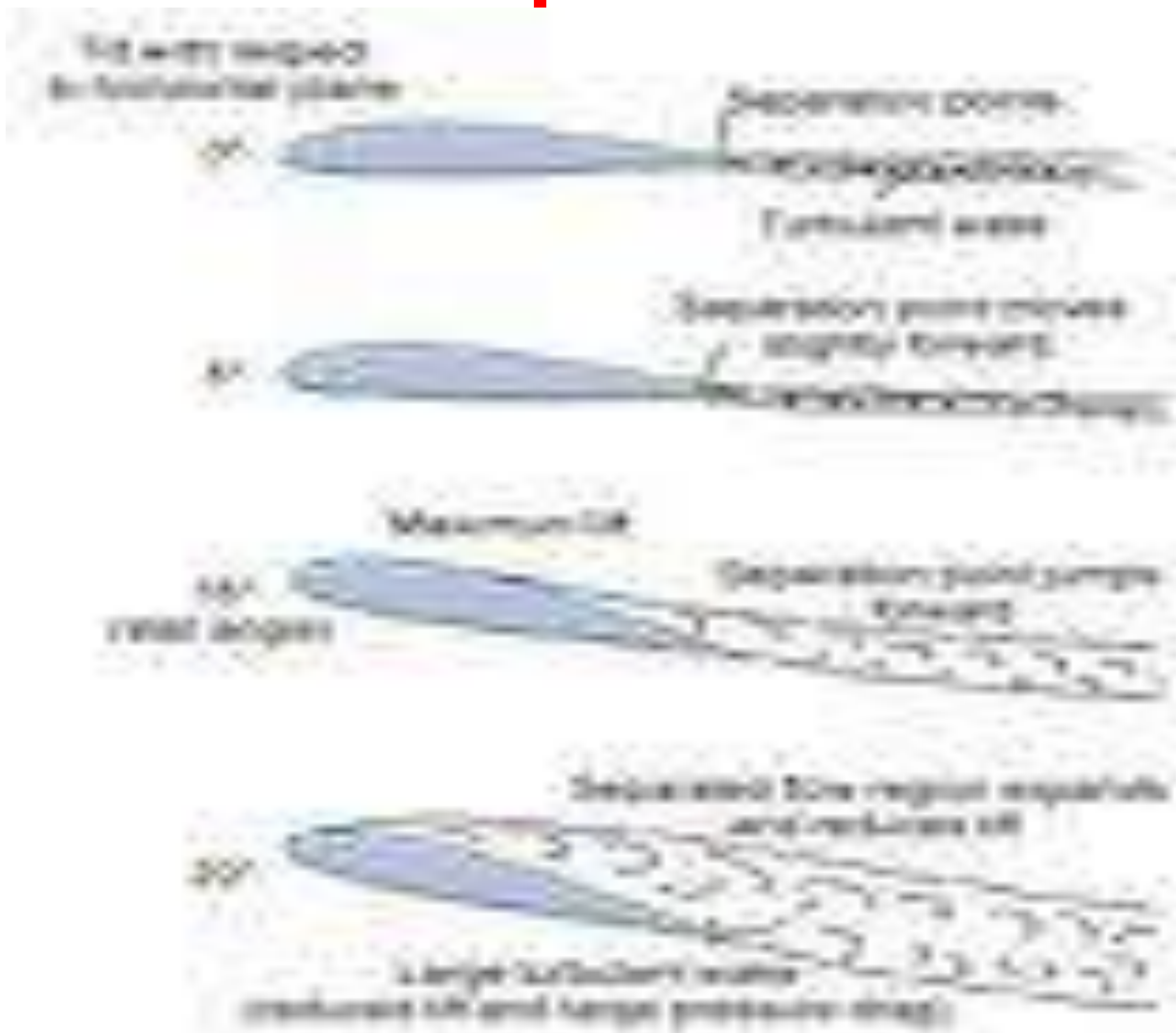
- Flow Separation at Reynolds = 100 000



What Happens When the Boundary Layer Separates?

- When the boundary layer separates, its displacement thickness increases sharply, which modifies the outside potential flow and pressure field.
- In the case of airfoils, the pressure field modification results in an increase in pressure drag, and if severe enough will also result in loss of lift and stall.
- Another effect of boundary layer separation is shedding vortices, known as Kármán vortex sheet. The shedding of the vortices then could cause vibrations in the structure
- Flow separation is the reason for wing stall at high angle of attack

Wing Stall Due to Boundary Layer Separation



Vortex Shedding due to Boundary Layer Separation

Effects of Boundary Layers on Flight

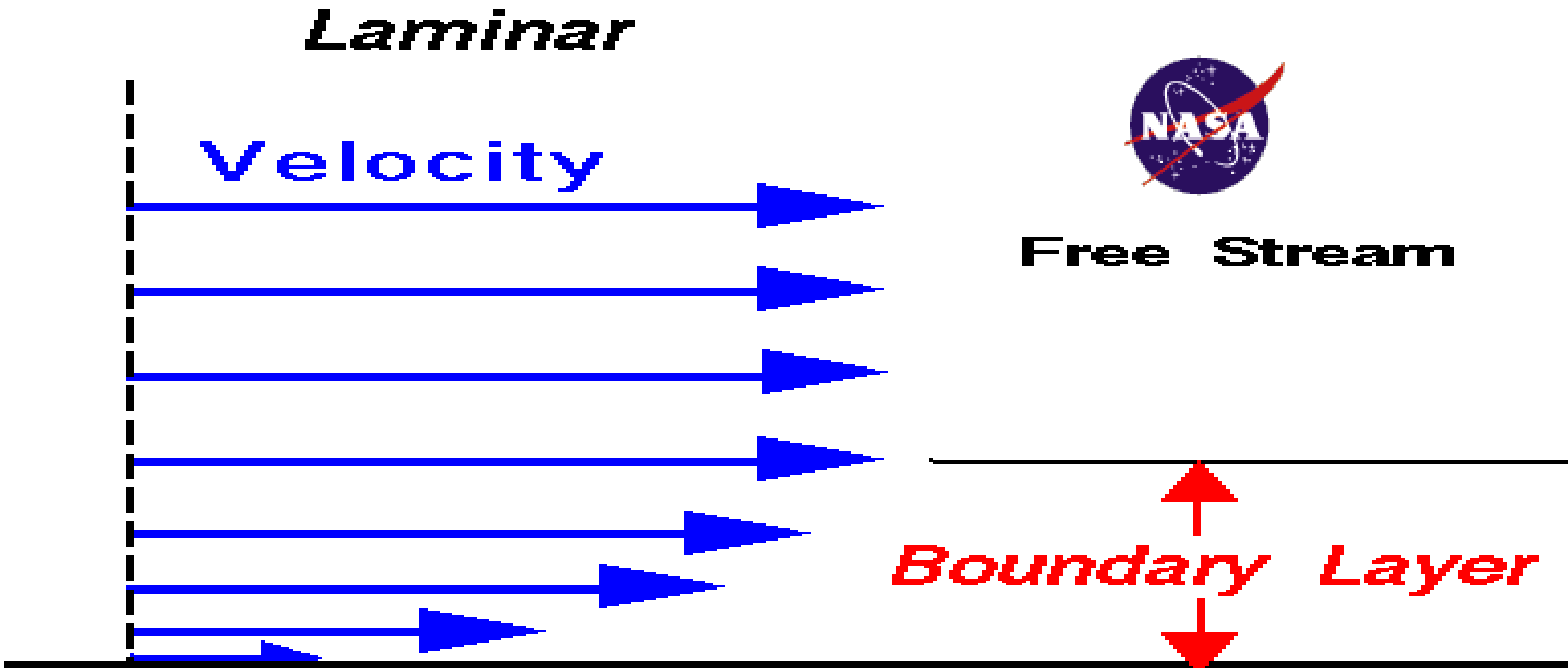
- In high-performance designs, such as sailplanes and commercial transport aircraft, much attention is paid to controlling the behavior of the boundary layer to minimize drag.
- Two effects have to be considered. First, the boundary layer adds to the effective thickness of the body, through the displacement thickness, hence increasing the pressure drag. Secondly, the shear forces at the surface of the wing which create skin friction drag.
- Of course, as mentioned in the ABOVE slides, boundary layer separation can become a serious flight stability issue due to its separation.

Types of Boundary Layers

- As you know, the boundary layers are the thin, sticky viscous region where the fluid is slowed down and thus viscous effects take place.
- Boundary layers may be either **laminar** (layered), or **turbulent** (disordered) depending on the value of the Reynolds number.
- The mathematical treatment of a laminar and turbulent boundary layer differ greatly

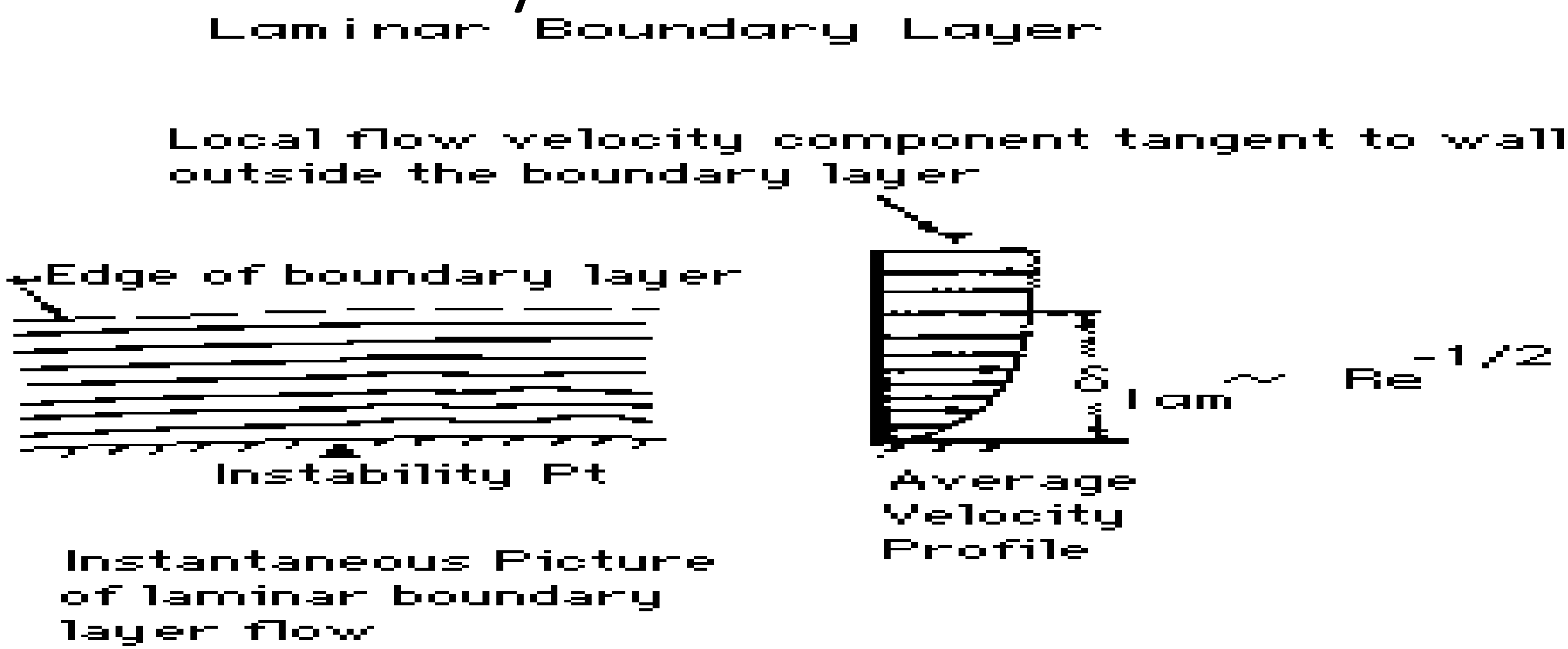
Laminar Boundary Layers

- For lower Reynolds numbers, the boundary layer is laminar and the streamwise velocity changes uniformly as one moves away from the wall

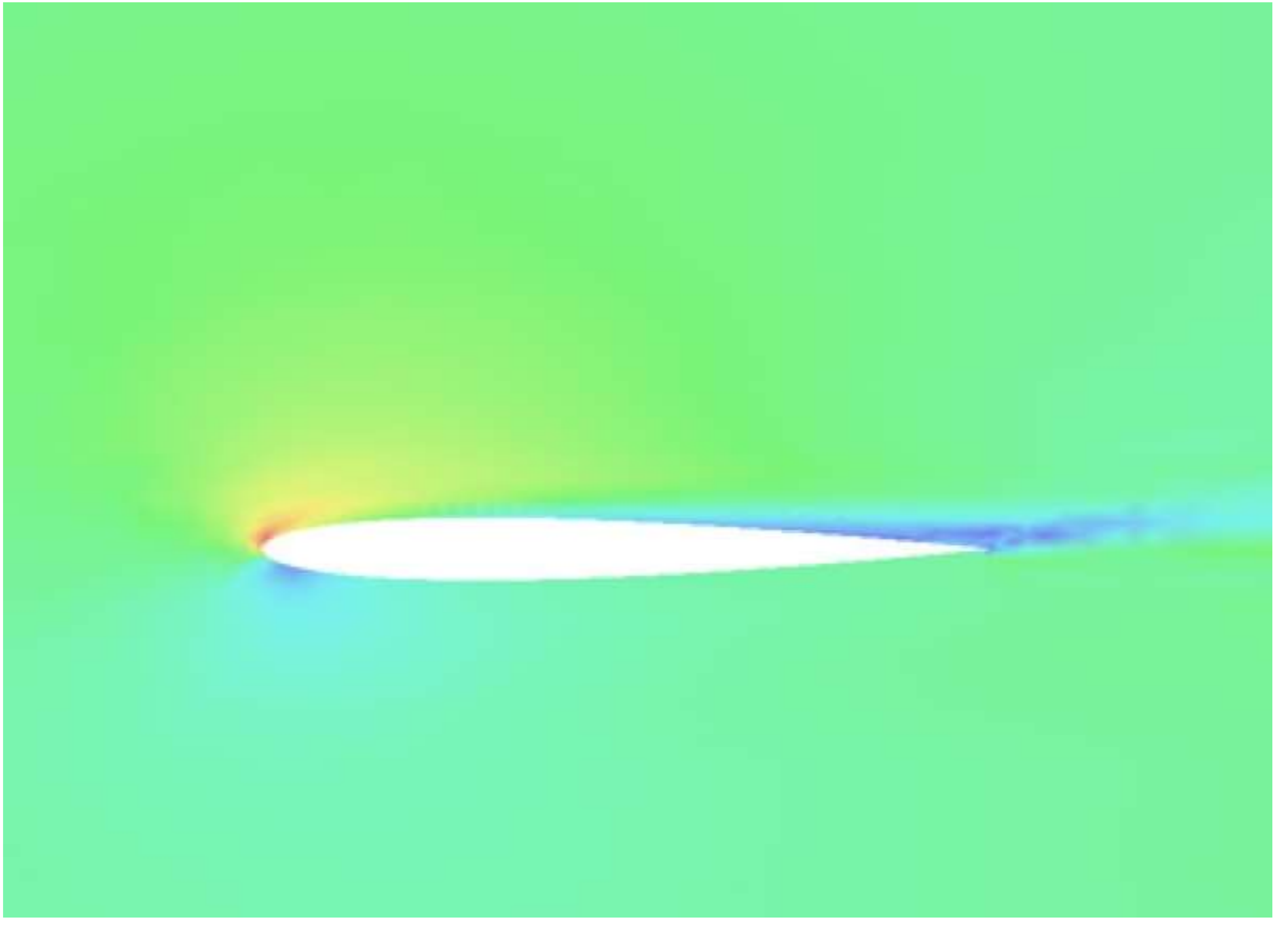


Laminar Boundary Layers

- In laminar flow, the fluid moves in smooth layers or lamina. There is relatively little mixing and consequently the velocity gradients are small and shear stresses are low.
- The thickness of the laminar boundary layer increases with distance from the start of the boundary layer and decreases with Reynolds number.



Laminar Boundary Layers

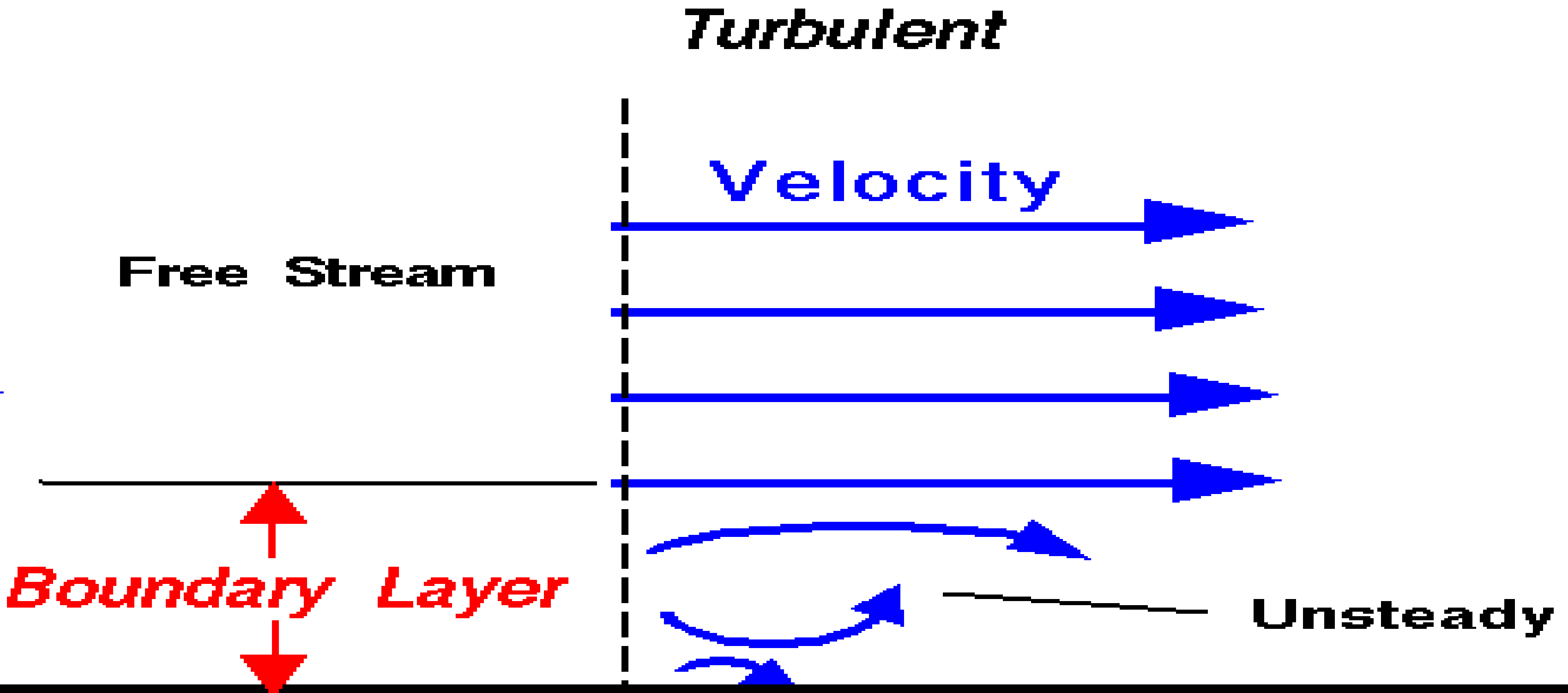


Laminar Flow and Laminar Boundary Layers

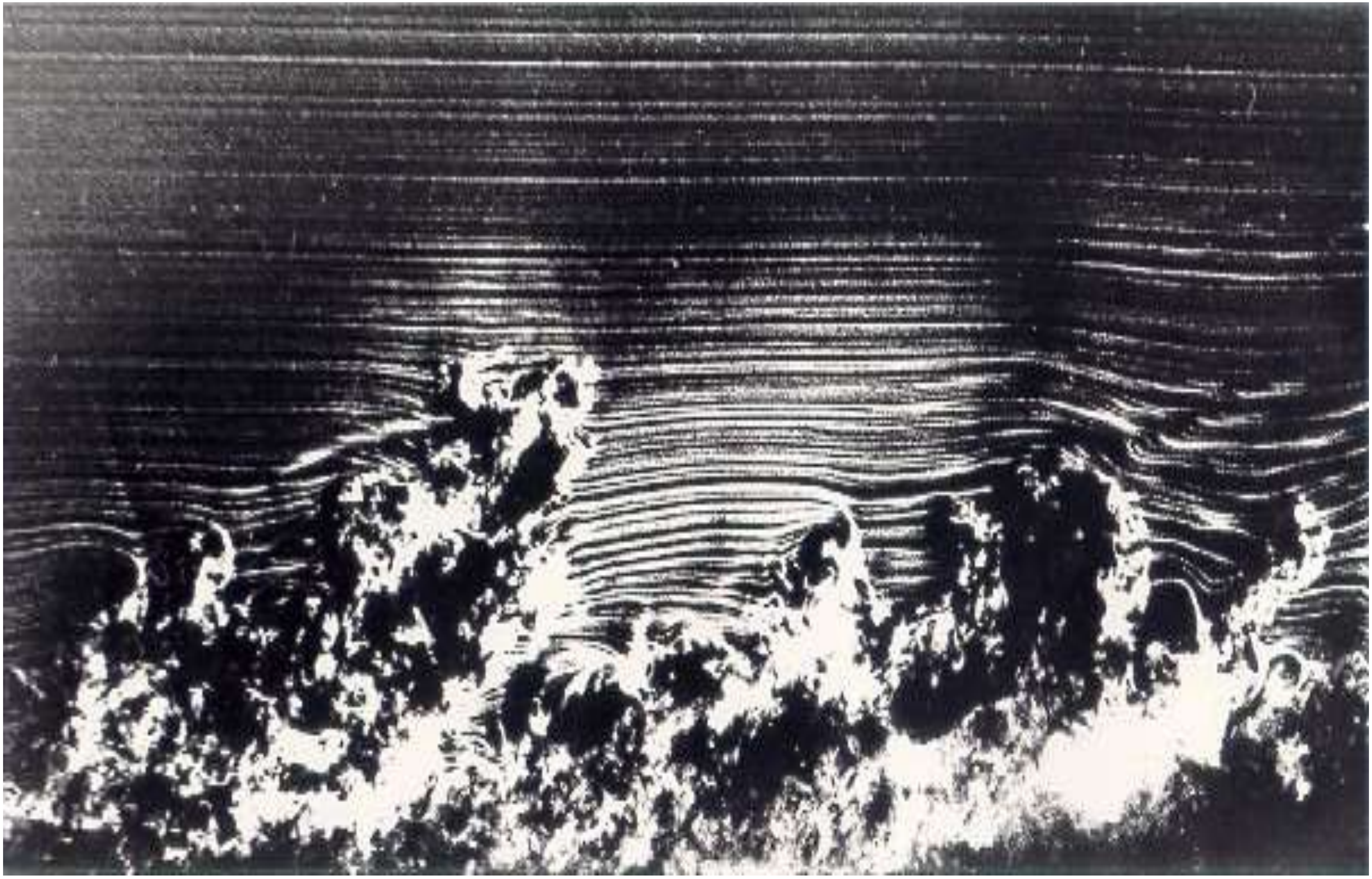
- At lower Reynolds numbers, such as those seen with model aircraft, it is relatively easy to maintain laminar flow. This gives low skin friction, which is desirable. However, the same velocity profile which gives the laminar boundary layer its low skin friction also causes it to be badly affected by adverse pressure gradients
- As the pressure begins to recover over the rear part of the wing chord, a laminar boundary layer will tend to separate from the surface. Such flow separation causes a large increase in the pressure drag

Turbulent Boundary Layers

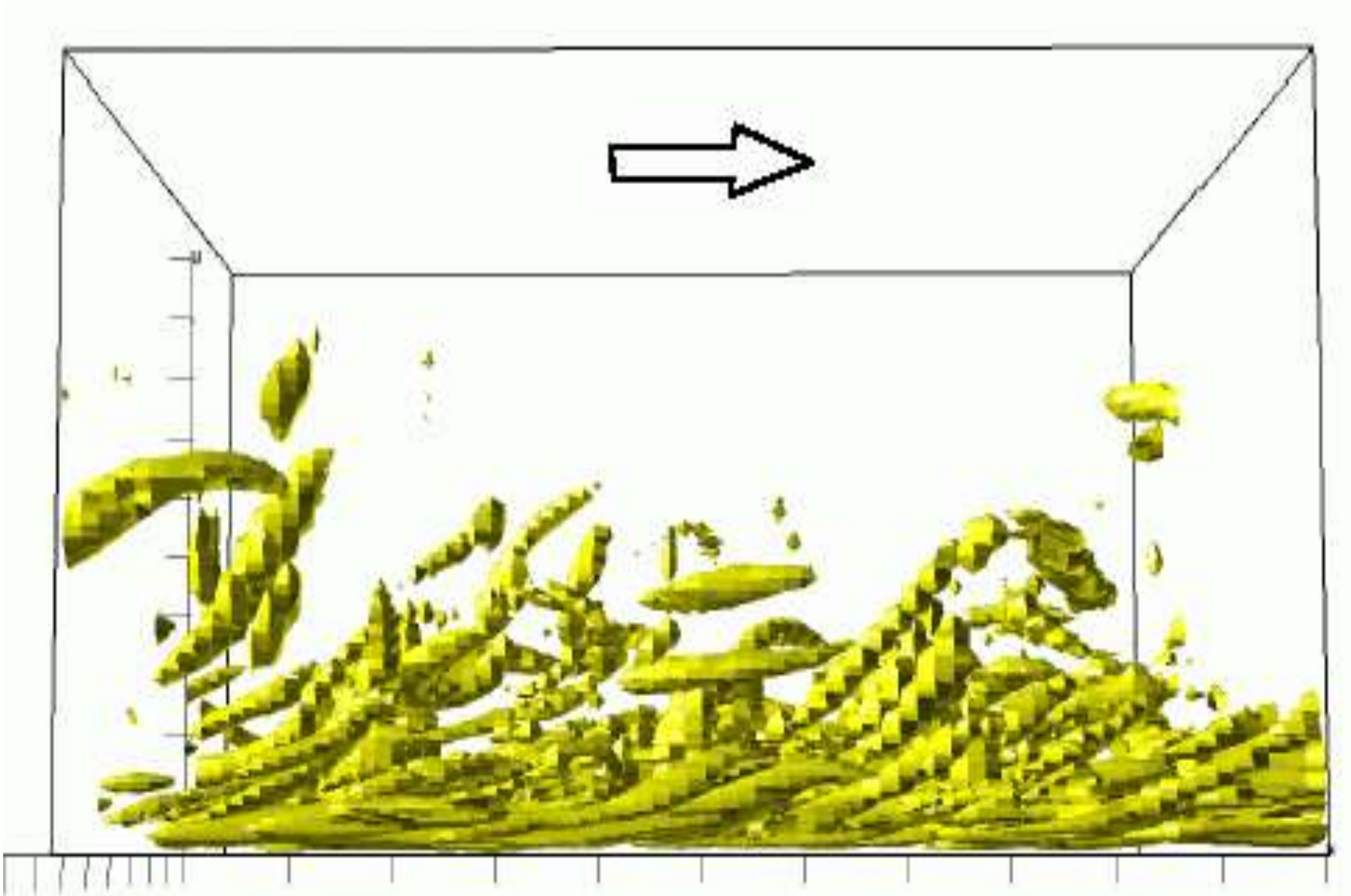
- For higher Reynolds numbers, the boundary layer is turbulent and the *streamwise velocity is characterized by unsteady (changing with time) swirling flows inside the boundary layer*.



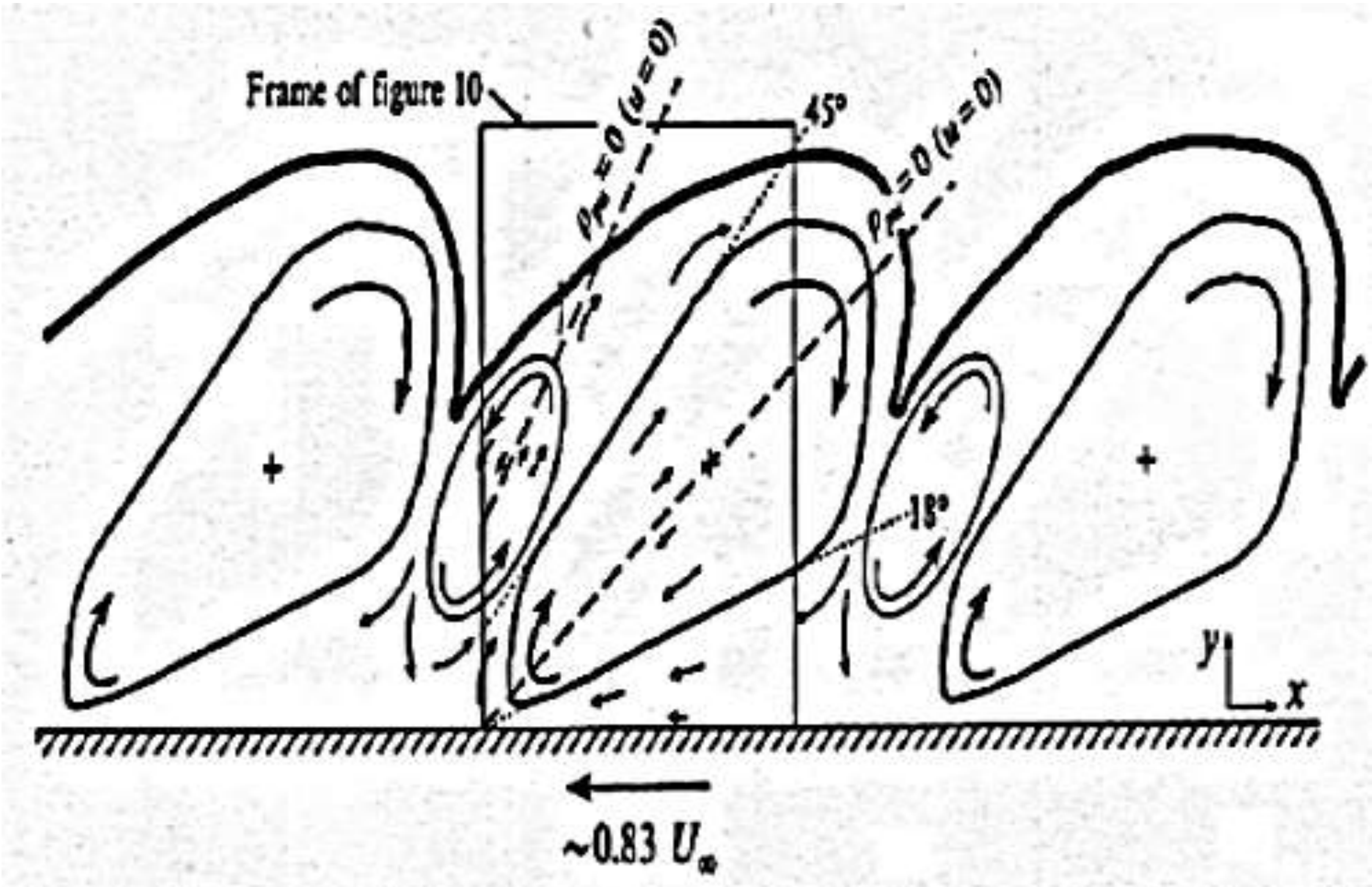
Turbulent Boundary Layer



Turbulent Boundary Layer

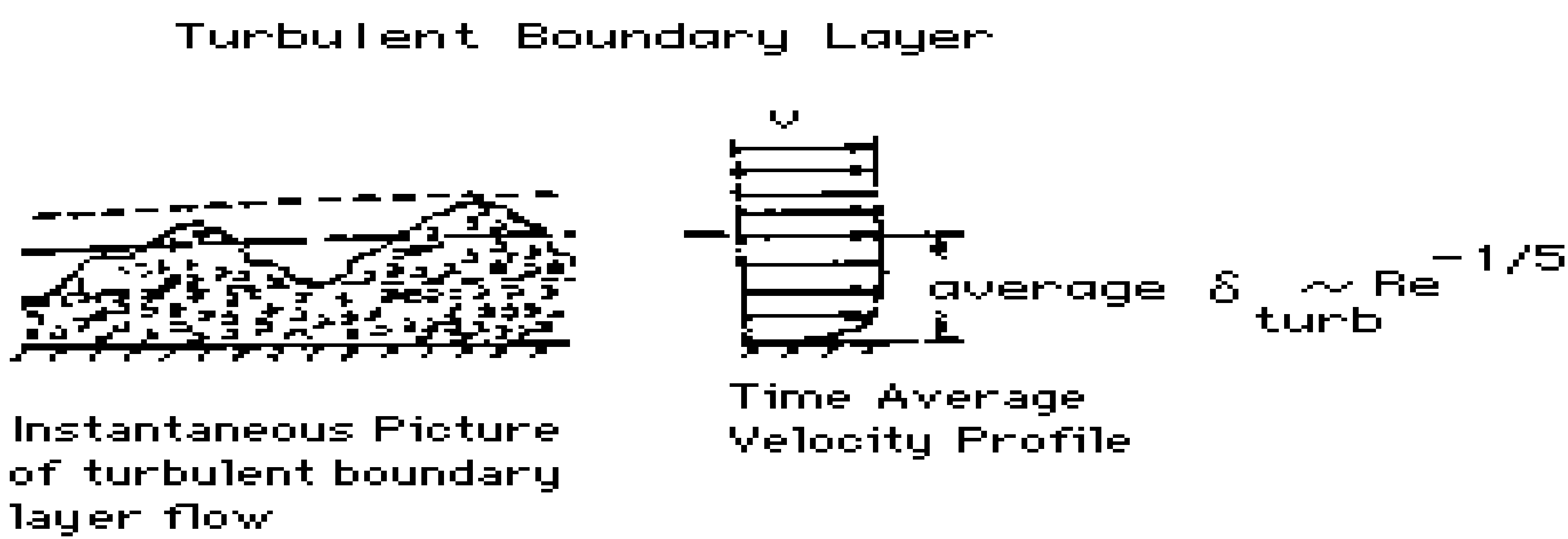


Turbulent Boundary Layer

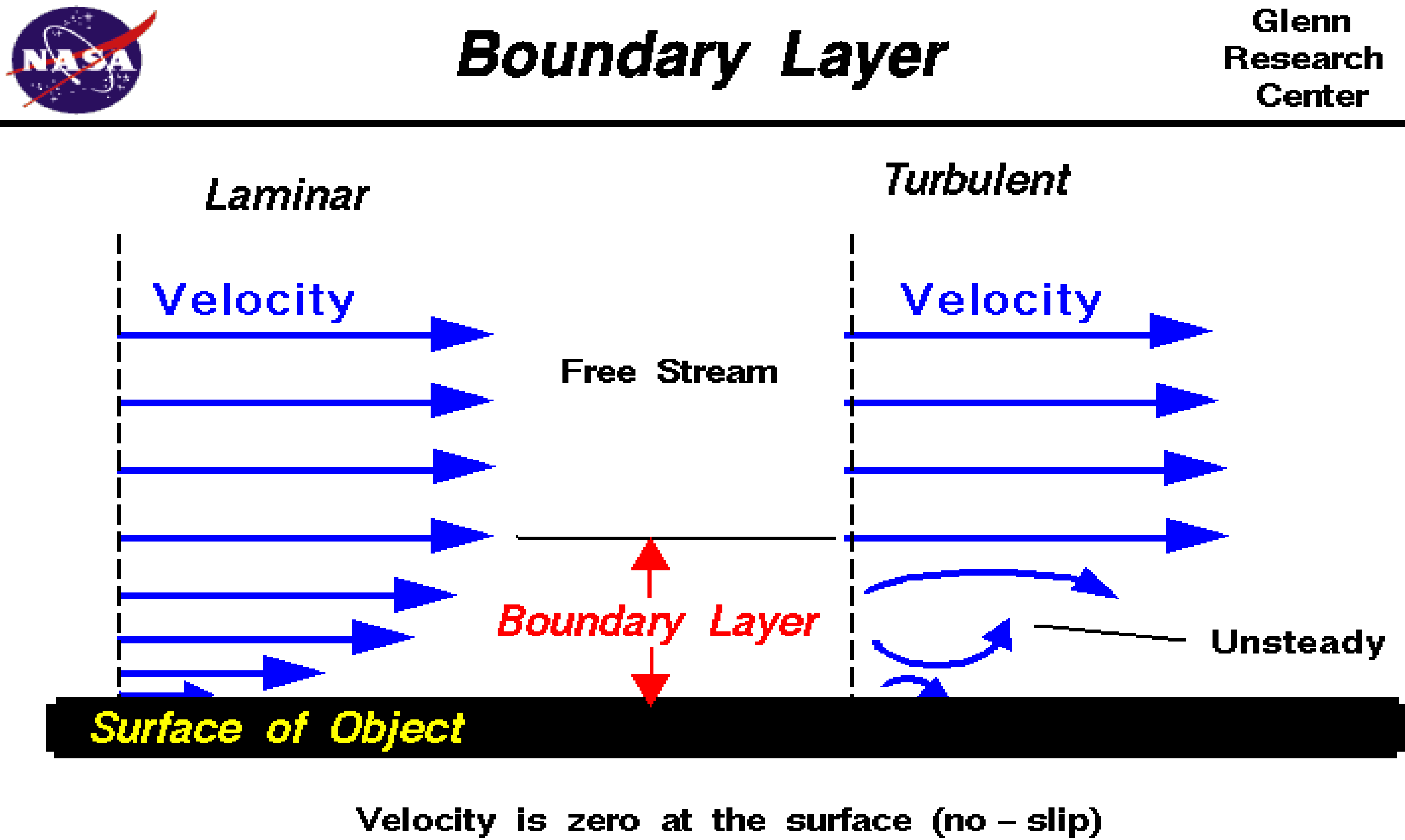


Turbulent Boundary Layers

- Turbulent boundary layer flow is characterized by unsteady mixing due to eddies at many scales. The result is ***higher shear stress at the wall, and a "fuller" velocity profile.***
- The wall shear stress is higher ***because the velocity gradient near the wall is greater. T***his is because of the more effective mixing associated with turbulent flow



Comparison of Boundary Layers
Turbulent - Laminar



Transition from Laminar Boundary Layer to Turbulent B.L.

- In reality, especially in fast motion dynamics, both a laminar turbulent layer, as well as a turbulent boundary layer will be present.
- There will also be a transitional boundary layer between the laminar and turbulent layers.

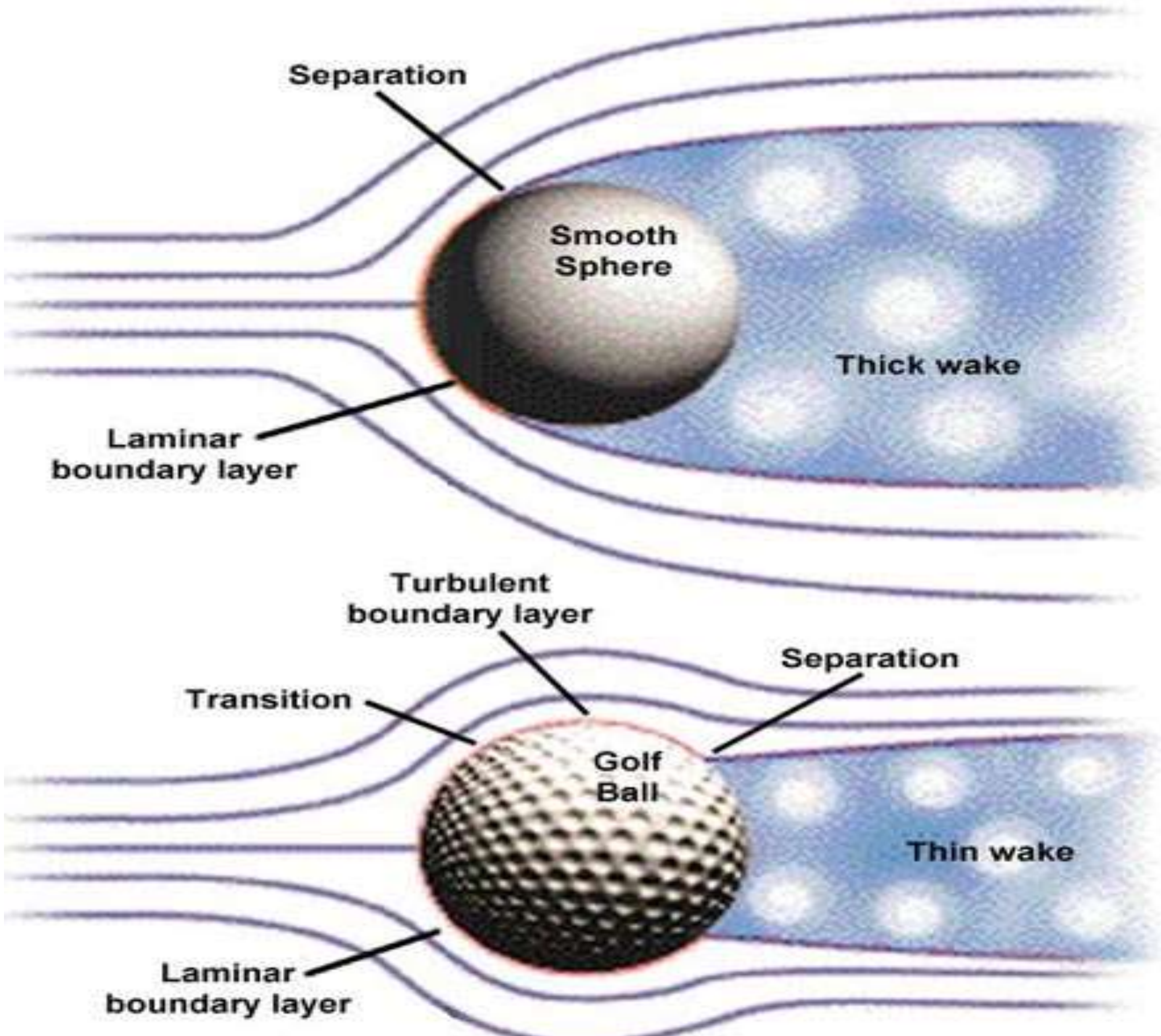
Comparison of Boundary Layers Turbulent - Laminar

- So, it seems as if a laminar boundary layer is better then a turbulent boundary layer, since there doesn’t seem to be adverse velocity gradients! Right??????

WRONGGGGGGGGGG!

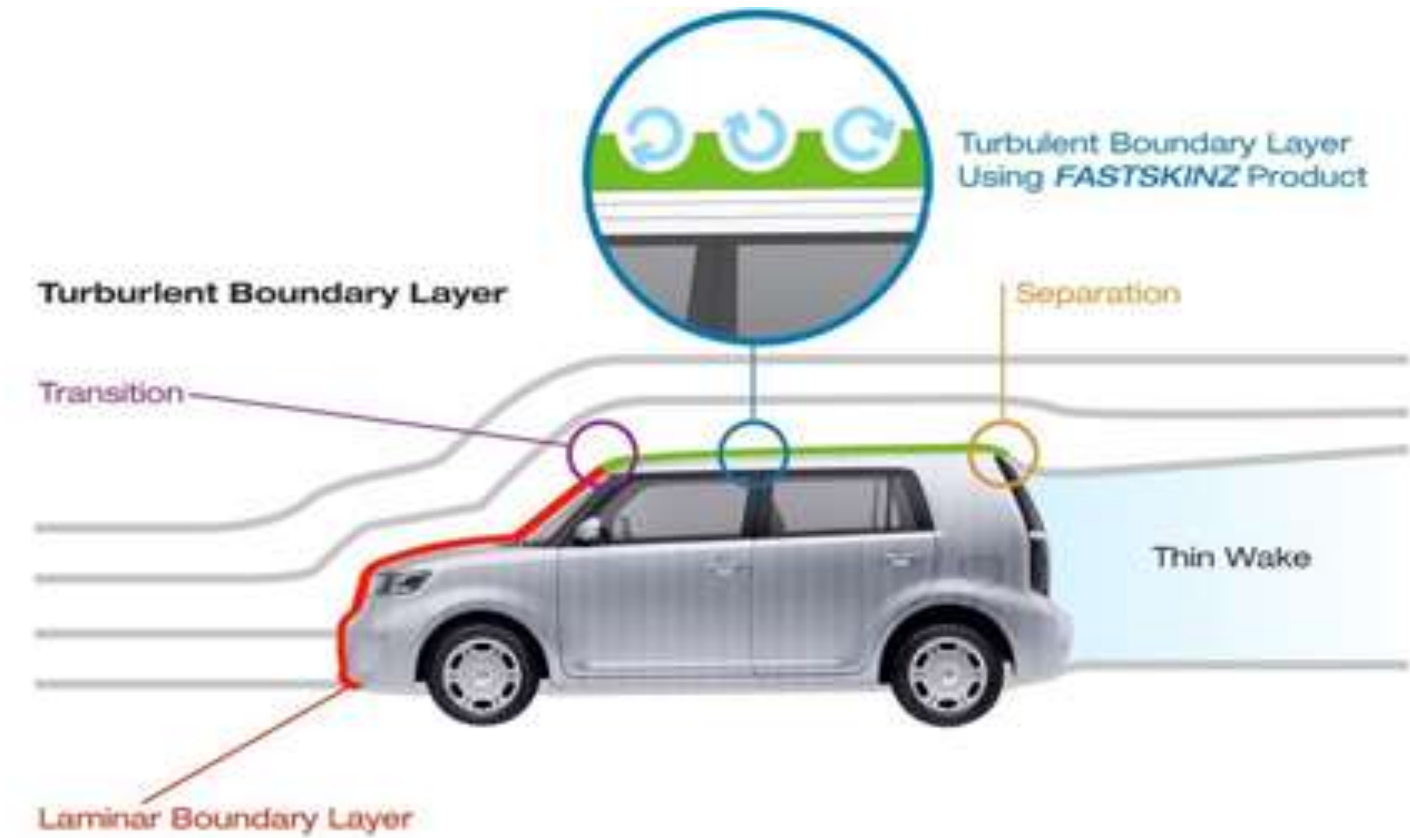
- Remember that a turbulent boundary layer (as predictable as it may be, it will not allow a flow separation to take place as easily as a laminar boundary layer.
- When the flow separation does take place, it happens further from the central axis of the body.

Comparison of Boundary Layers
Turbulent - Laminar

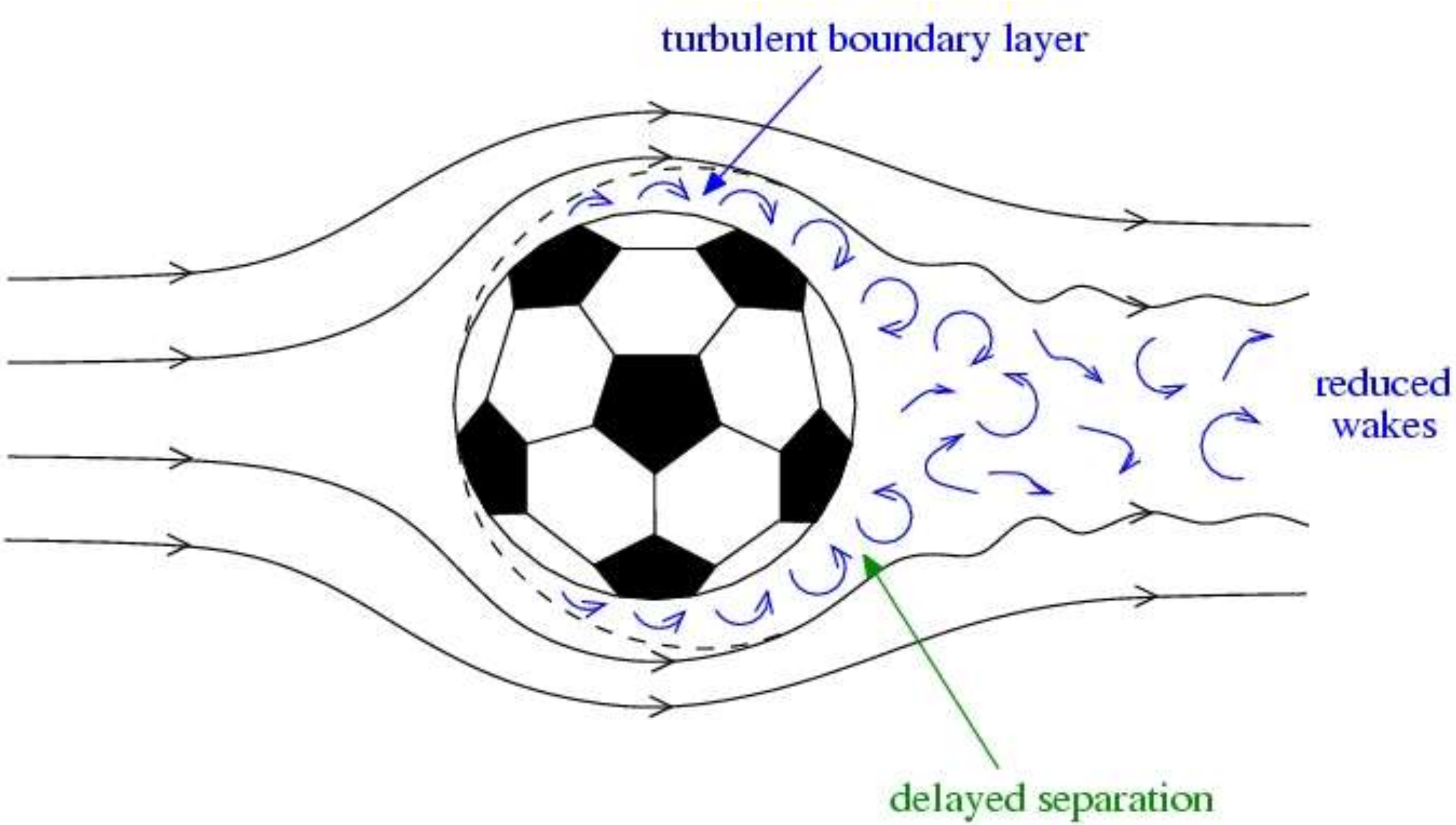


Comparison of Boundary Layers Turbulent - Laminar

- So does turbulent boundary layer work only for golf balls? ***How about something more common?***



How About Boundary Layers and Soccer?



Comparison of Boundary Layers Turbulent - Laminar

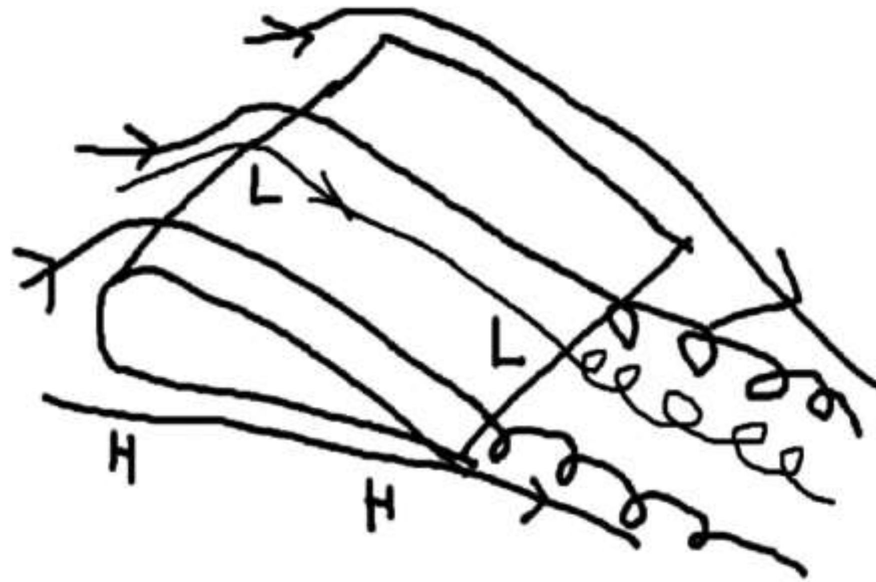
- How about our good childhood friend Frisby?

The rotation of a frisbee makes the boundary layer turbulent even with moderate forward speed of the frisbee, which makes the frisbee into an efficient wing with L/D allowing a long distance of travel under moderate initial speed

For a rapidly rotating frisbee, the boundary layer can become turbulent which can drastically improve the lift/drag quotient and improve flight characteristics.

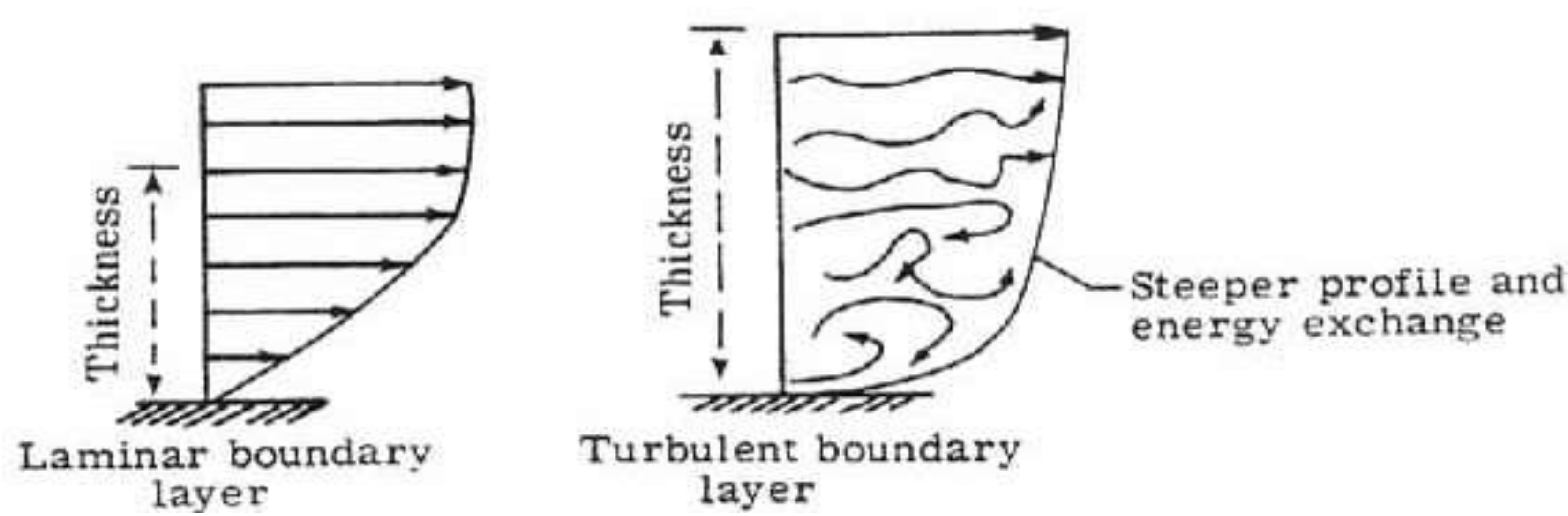
Properties of a Turbulent Boundary Layer in Flight

- Viscous flow with a laminar boundary layer separates at the crest and gives poor lift and large drag.
- The perturbed flow **does not separate at the crest** because the **<u>boundary layer is turbulent</u>**



Why Do We Want a Turbulent Boundary Layer on an Aircraft?

- The fuller velocity profile of the turbulent boundary layer allows it to sustain the adverse pressure gradient without separating. Thus, although the skin friction is increased, overall drag is decreased.



Properties of a Turbulent Boundary Layer in Flight

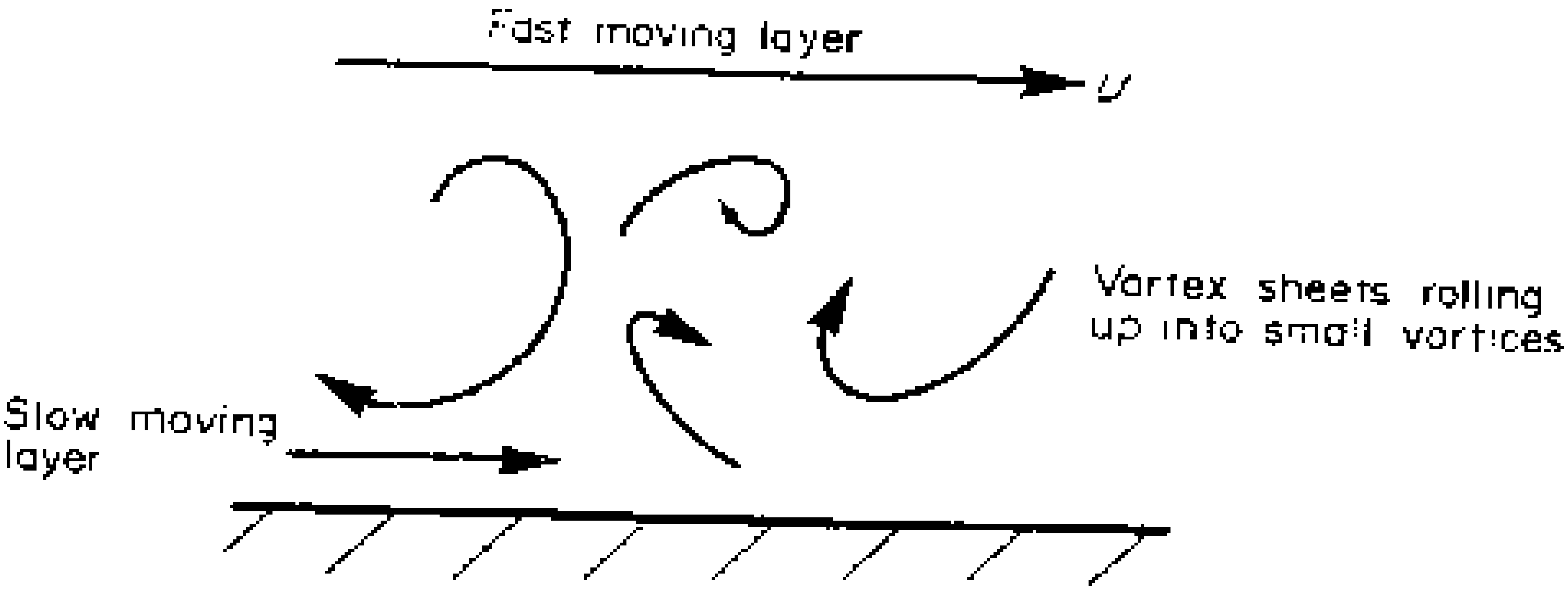
- So in actual life in viscous flow situations, we want a turbulent boundary layer, just as we want turbulent flow for high speed aircraft and spacecraft.
- What determines if the boundary layer is turbulent (which is good) or laminar (which is bad) is the

  Reynolds number

  SO HIGHER REYNOLDS NUMBER FLIGHTS ARE BETTER FOR STABILITY, SINCE HIGH REYNOLDS NUMBERS MEAN TURBULENT FLOW AND A TURBULENT BOUNDARY LAYER

How Does a Turbulent Boundary Layer Form?

- In laminar flow, The viscous shear stresses have held the fluid particles in a constant motion within layers. They become small as the boundary layer increases in thickness and the velocity gradient gets smaller. Eventually they are no longer able to hold the flow in layers and the fluid starts to rotate.

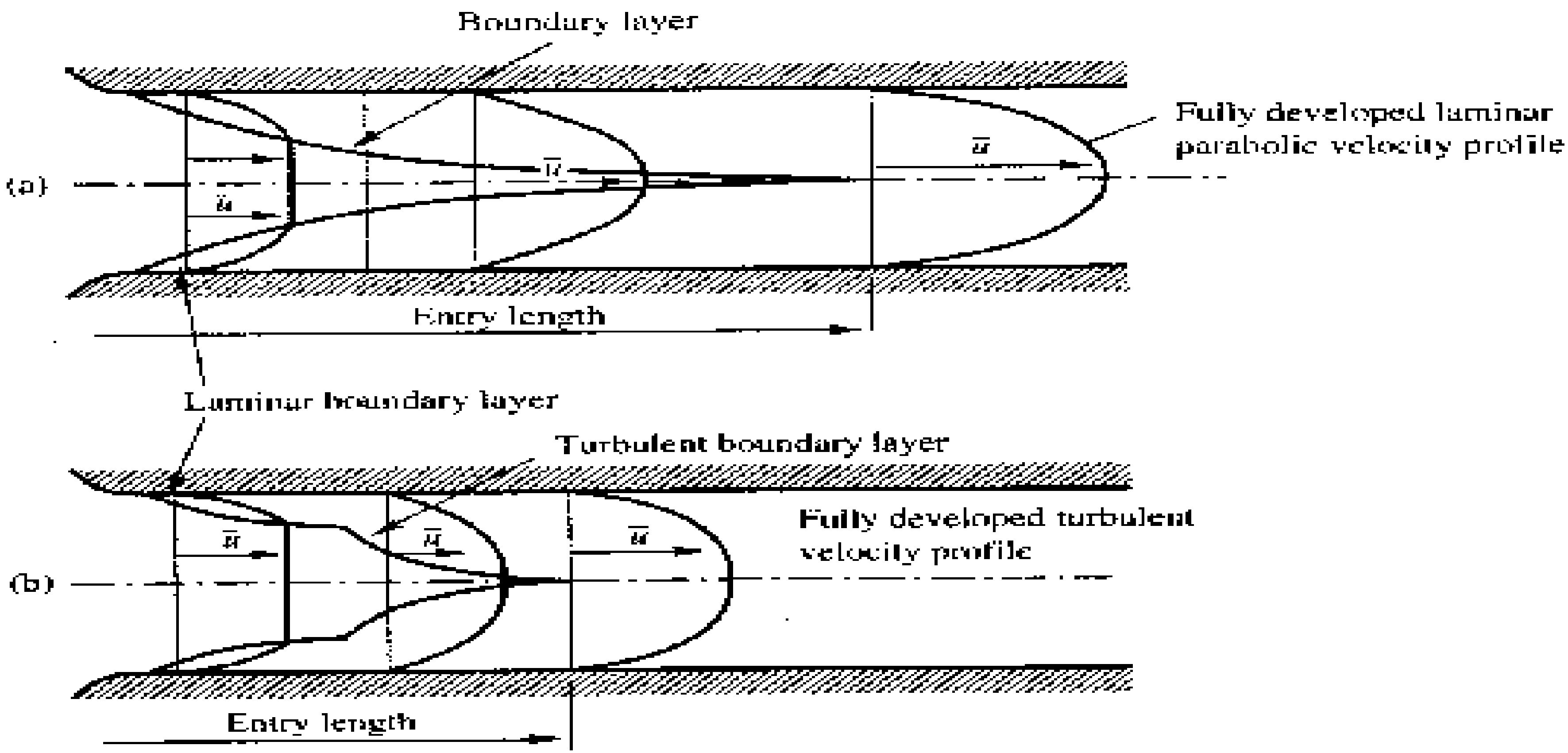


This causes the fluid motion to rapidly becomes turbulent

How Does a Turbulent Boundary Layer Form?

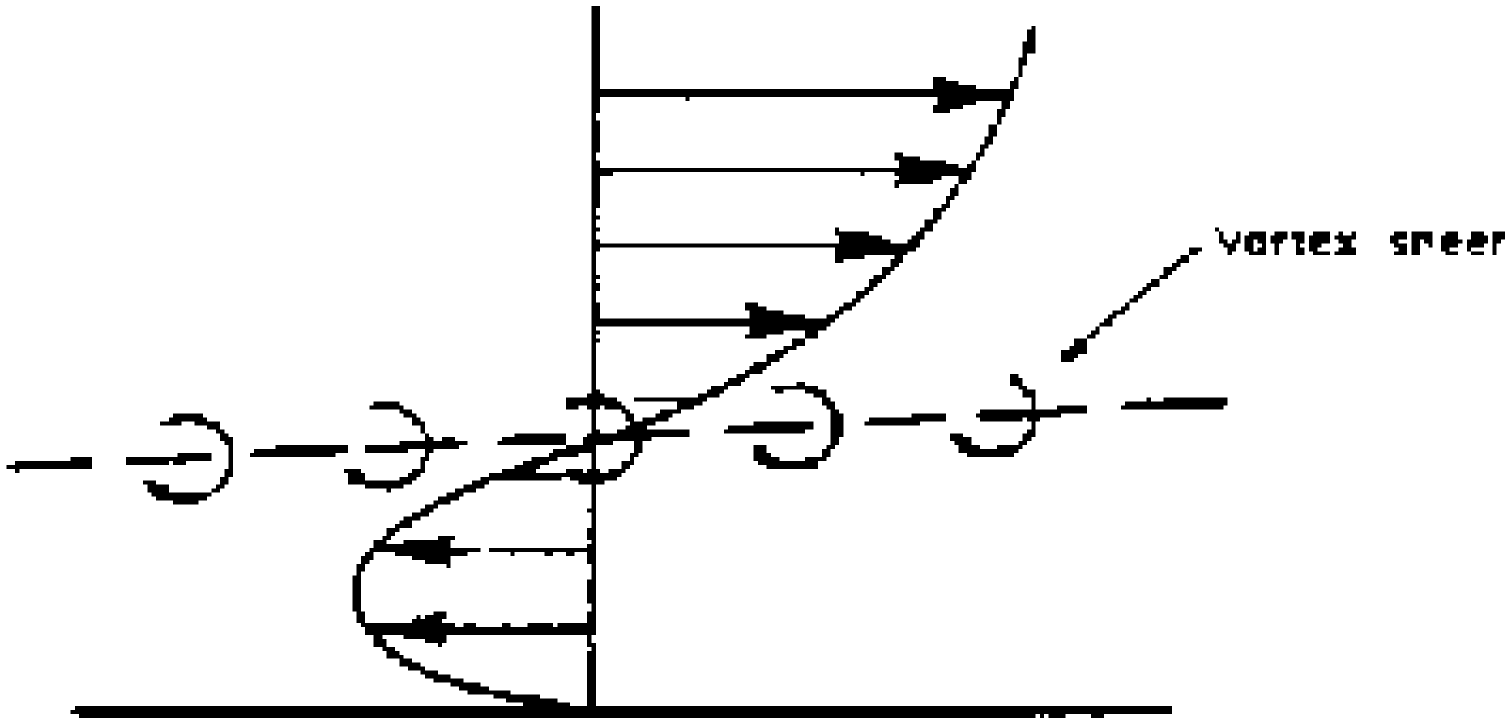
- Fluid from the fast moving region moves to the slower zone transferring momentum and thus maintaining the fluid by the wall in motion. Conversely, slow moving fluid moves to the faster moving region slowing it down. The net effect is an increase in momentum in the boundary layer.

  We call the part of the boundary layer the **turbulent boundary layer**.



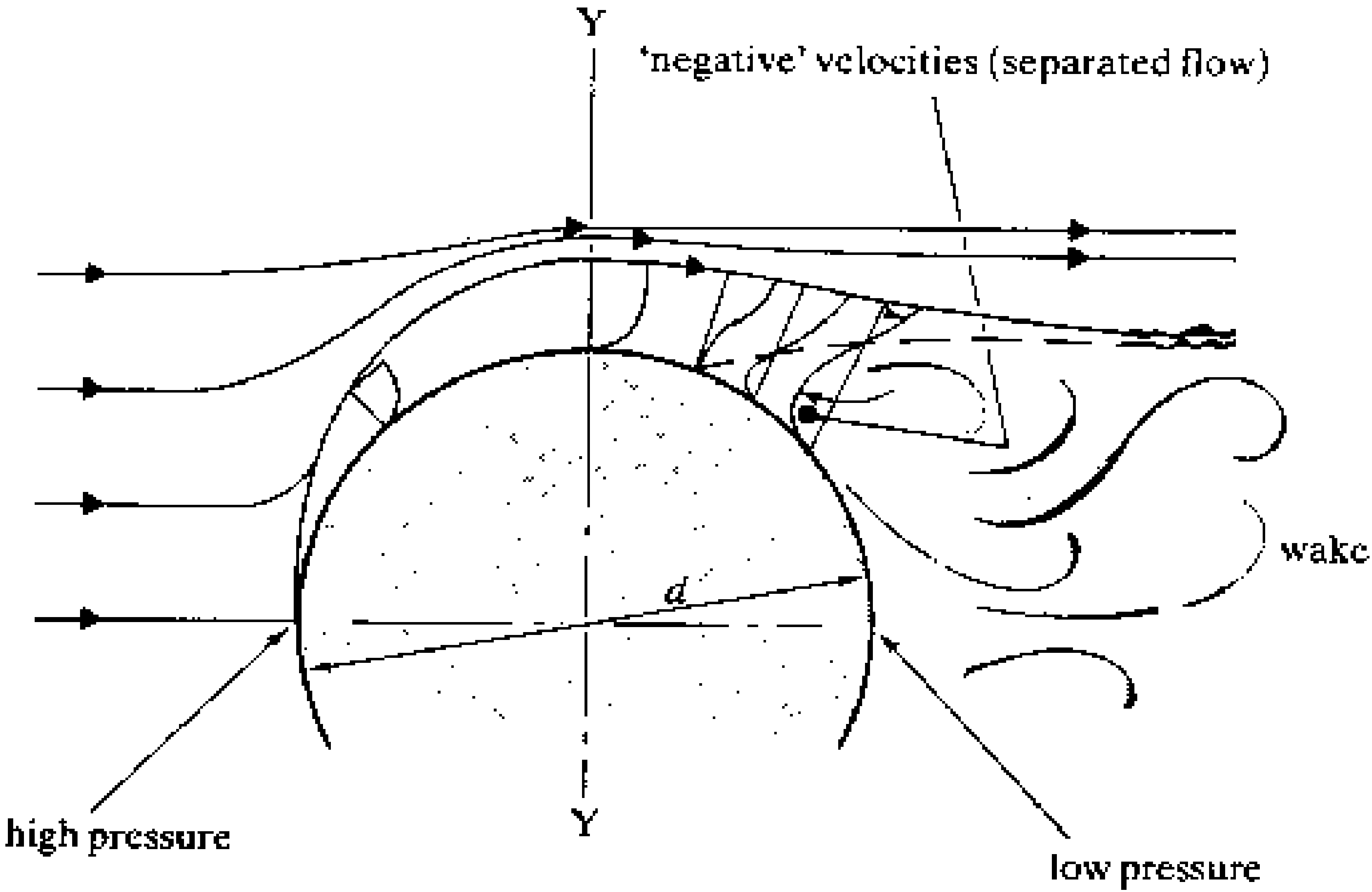
Boundary Layer Separation

- At the edge of the separated boundary layer, where the velocities change direction, a line of vortices occur (known as a **<u>vortex sheet</u>**). This happens because fluid to either side is moving in the opposite direction.



This boundary layer separation and increase in the turbulence because of the vortices results in very large energy losses in the flow

Boundary Layer Separation



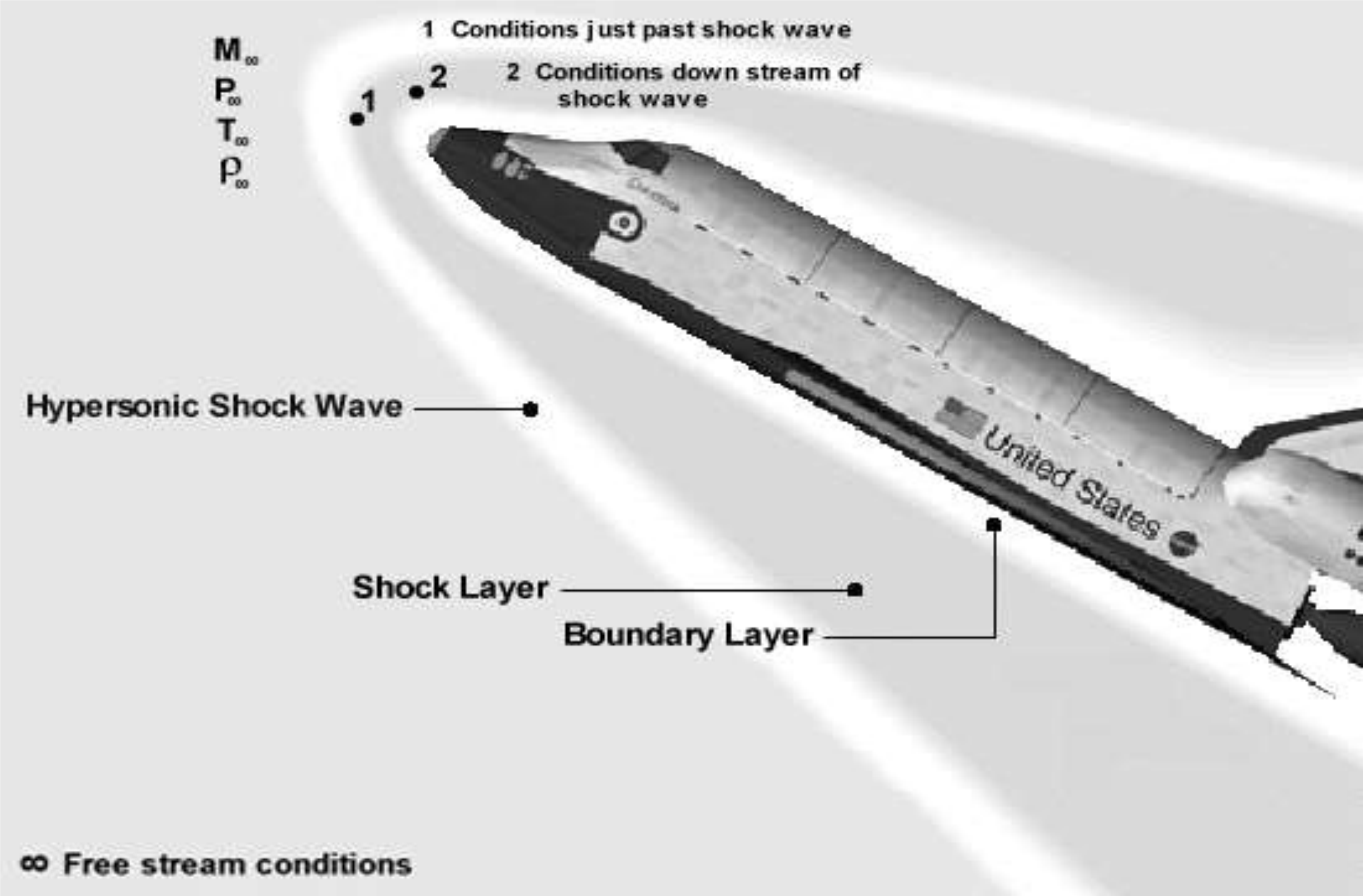
Some Usage of Turbulent Boundary Layers on Aircraft

- Turbulent flow and turbulent boundary layers are especially welcome on high speed aircraft. In order to introduce turbulent boundary layers, leading edge extension (LEX) are used to delay flow separation

Vortex Generators on Aircraft

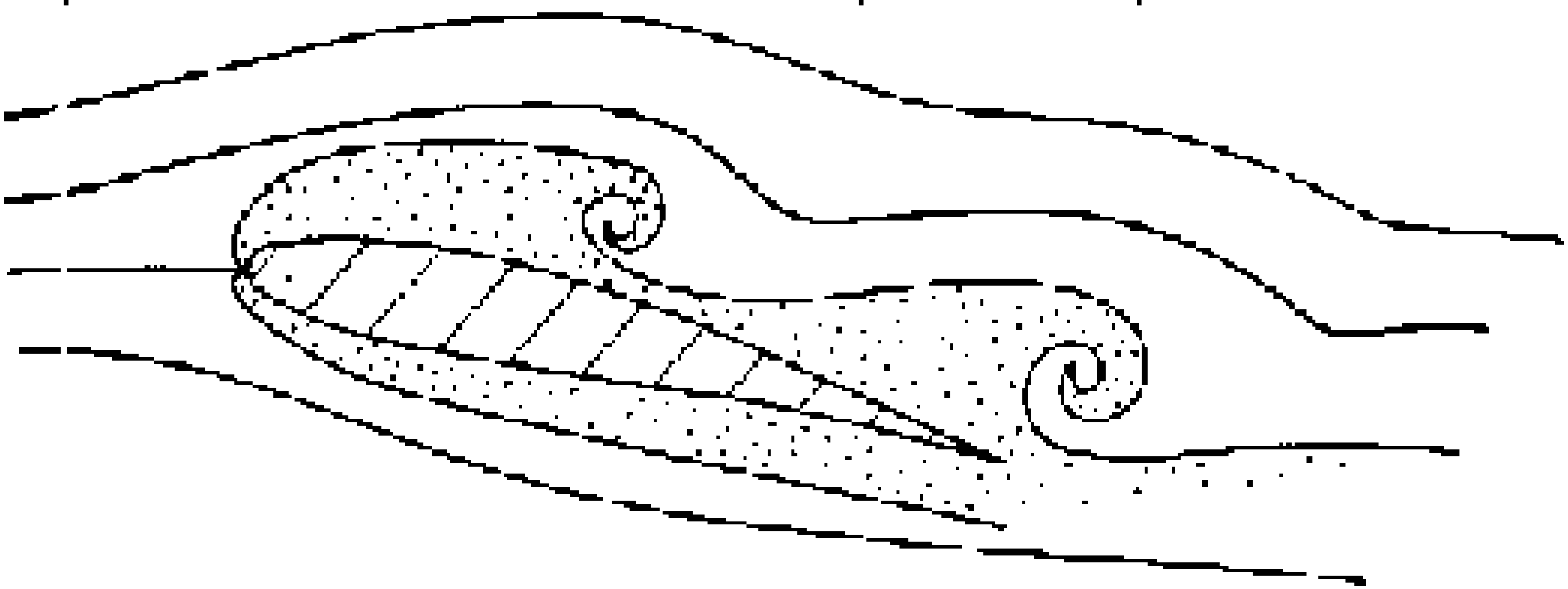
- Vortex generators are often placed along the outer portion of a wing in order to promote a turbulent boundary layer that adds forward momentum to the flow. As in the case of the golf ball, this turbulent boundary layer helps the flow overcome an adverse pressure gradient and remain attached to the surface longer than it would otherwise.

Boundary Layers on the Space Shuttle



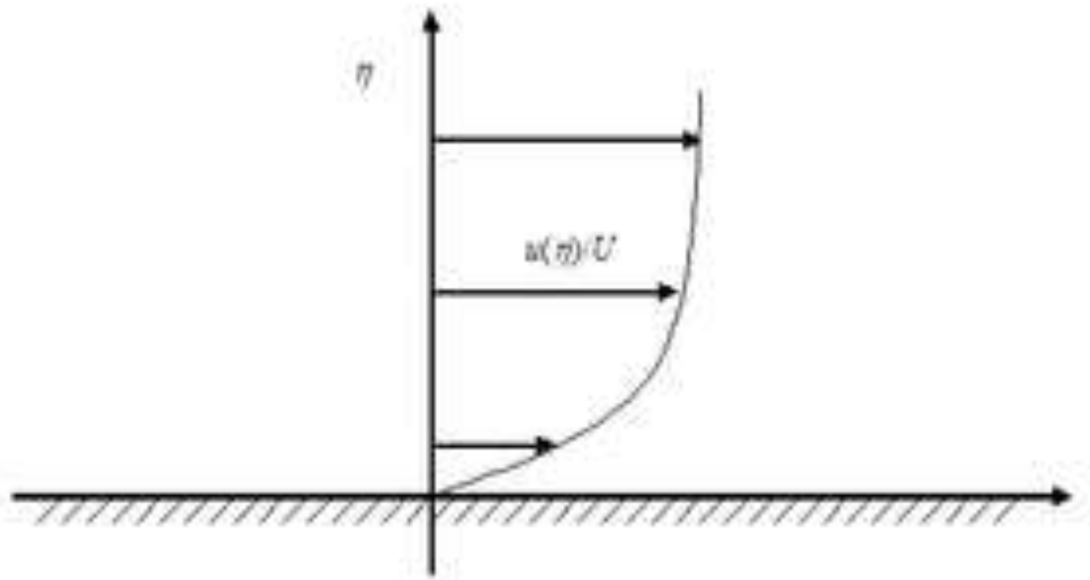
How to Prevent Stalling?

- If the angle of the wing becomes too great and boundary layer separation occurs on the top of the aerofoil ,the pressure pattern will change dramatically. This phenomenon is known as **stalling**. The separation of the boundary layer can cause stalling due to huge loss of momentum and energy instantaneously.
- When stalling occurs, all, or most, of the 'suction' pressure is lost, and the plane will suddenly drop from the sky! The only solution to this is to put the plane into a dive to regain the boundary layer. A transverse lift force is then exerted on the wing, which gives the pilot some control and allows the plane to be pulled out of the dive.



Laminar Boundary Layer of a Flow Over a Flat Plate (Blasius Flow)

- A **Blasius boundary layer**, in physics and fluid mechanics, describes the steady two-dimensional boundary layer that forms on a semi-infinite plate which is held parallel to a constant unidirectional flow *U*.



A **schematic diagram of the Blasius flow profile**

Laminar Boundary Layer of a Flow Over a Flat Plate (Blasius Flow)

- the semi-infinite plate has no natural length scale *L* and so the steady, incompressible, two-dimensional boundary-layer equations for continuity and momentum are:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Blasius Solution

$$\delta(x) \approx \left(\frac{\nu x}{U}\right)^{1/2}.$$

This suggests adopting the similarity variable

$$\eta = \frac{y}{\delta(x)} = y\left(\frac{U}{\nu x}\right)^{1/2}$$ $u = Uf'(\eta)$.

It proves convenient to work with the stream function ψ, in which case

$\psi = (\nu U x)^{1/2} f(\eta)$

and on differentiating, to find the velocities, and substituting into the boundary-layer equation we obtain the Blasius equation

$$f''' + \frac{1}{2} f f'' = 0$$

$f = f' = 0$ on $\eta = 0$ ANI $f' \to 1$ $\eta \to \infty$

Laminar Boundary Layer of a Incompressible Flow Over a Flat Plate (Blasius Flow)

Local skin friction coefficient:

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

Integrated friction drag coefficient:

$$C_f = \frac{1.328}{\sqrt{\mathrm{Re}_c}}$$

Boundary-layer thickness:

$$\delta = \frac{5.0x}{\sqrt{\mathrm{Re}_x}}$$

Displacement thickness:

$$\delta^* = \frac{1.72x}{\sqrt{\mathrm{Re}_x}}$$

Momentum thickness:

$$\theta = \frac{0.664x}{\sqrt{\mathrm{Re}_x}}$$

Turbulent Boundary Layer of a Flow Over a Flat Plate

- Using the standard Boundary Layer equations, the solution for the boundary layer of a turbulent incompressible flow over a flat plate:

$$\delta = \frac{0.37x}{\mathrm{Re}^{\frac{1}{5}}} \qquad C_f = \frac{0.074}{\mathrm{Re}^{\frac{1}{5}}}$$

Summary of General Boundary Layer Equations

Continuity:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

x momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

y momentum:

$$\frac{\partial p}{\partial y} = 0$$

Energy:

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{dp_e}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2$$

These equations are subject to the boundary conditions:

At the wall: $y = 0 \quad u = 0 \quad v = 0 \quad h = h_w$

At the boundary-layer edge: $y \to \infty \quad u \to u_e \quad h \to h_e$

Inherent in the above boundary-layer equations are the assumptions that $\delta \ll c$, Re is large, and M_∞ is not inordinately large.

Boundary Layer Example Problem

- Consider a flat plate at zero angle of attack in an airflow at standard sea level conditions. The chord length of the plate (distance from the leading edge to the trailing edge is) 2 meters. The planform area of the plate is 40 m^2. Assume the wall temperature is the adibatic wall temperature Taw.
- Calculate the friction drag on the plate when the freestream velocity is 100 m/s:

  a) Laminar Flow and b) Turbulent Flow