Compressible Flow

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What is Compressible Flow?

- **Compressible Flow** is a type of flow in which the density can not be treated as constant.
- For most compressible flow situations, the speed of the flow will be either very close to the speed of sound or it will be much more faster than the speed of sound.
- Compressible flows are high energy flows. Thus, temperature changes and energy changes play an important role in compressible flow.

Speed of Sound in Compressible Flow

- Hence, the speed of sound plays an important role in compressible flow. Flows reaching or surpassing the speed of sound can be solved as compressible flow.
- Speed of Sound in an Ideal Gas is Given By the equation below and as it can be seen, it is mainly a function of the Temperature of the Freestream Flow.

$$a = \sqrt{\gamma R T}$$

Mach Number and the Speed of Flow

- Mach number denotes the velocity of the flow in relation to the speed of sound.
- Mach number 1 is the speed of sound for that medium.
- Most flight regimes fly less than Mach 1 today, such as commercial aircraft.

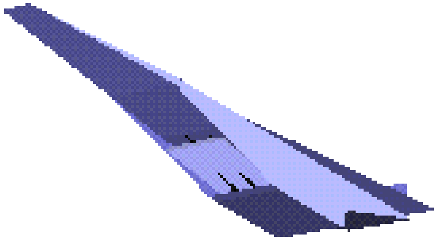
$$M = \frac{V}{a}$$

$$\begin{aligned} &\text{Subsonic if } M < 1 \\ &\text{Sonic if } M = 1 \\ &\text{Supersonic if } M > 1 \end{aligned}$$

Mach Number

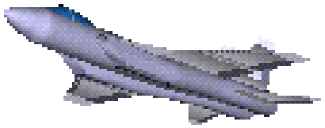
Glenn Research Center

$$\text{ratio} = \frac{\text{Object Speed}}{\text{Speed of Sound}} = \text{Mach Number}$$



Hypersonic

Mach > 5.0



Supersonic

Mach > 1.0

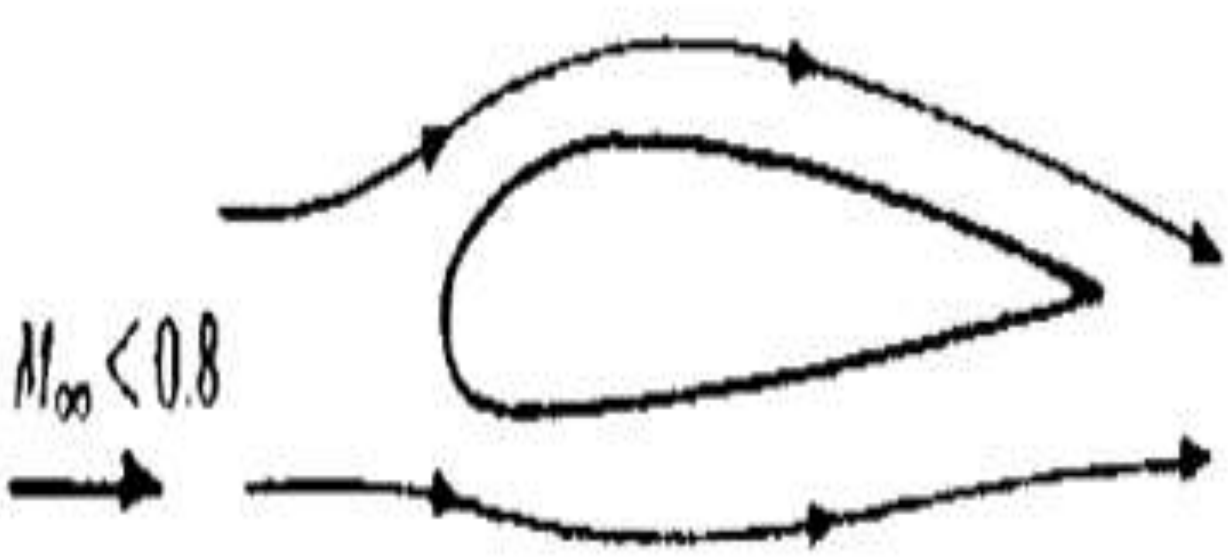
Transonic

Mach = 1.0

Subsonic

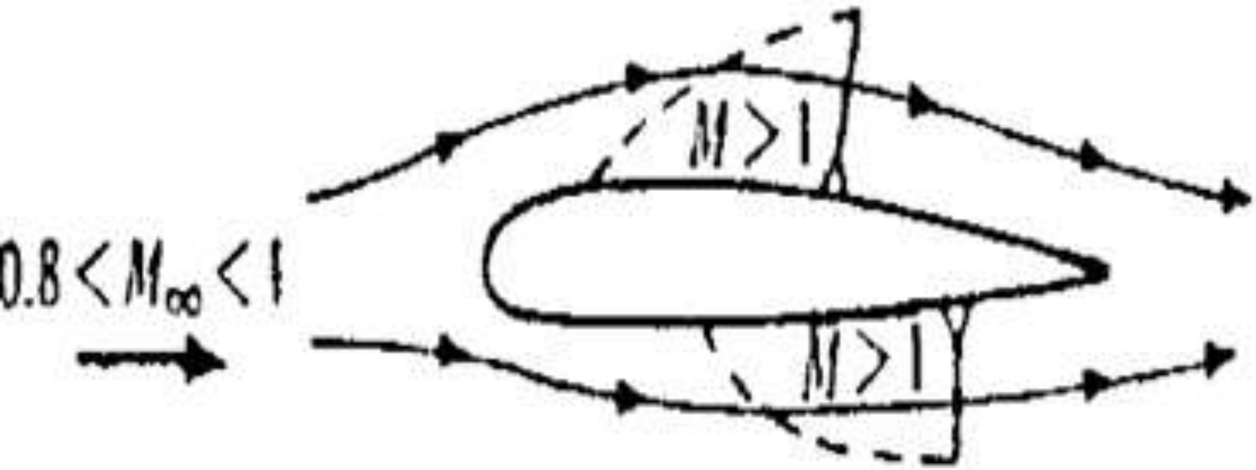
Mach < 1.0

Mach Number and the Flight Regime

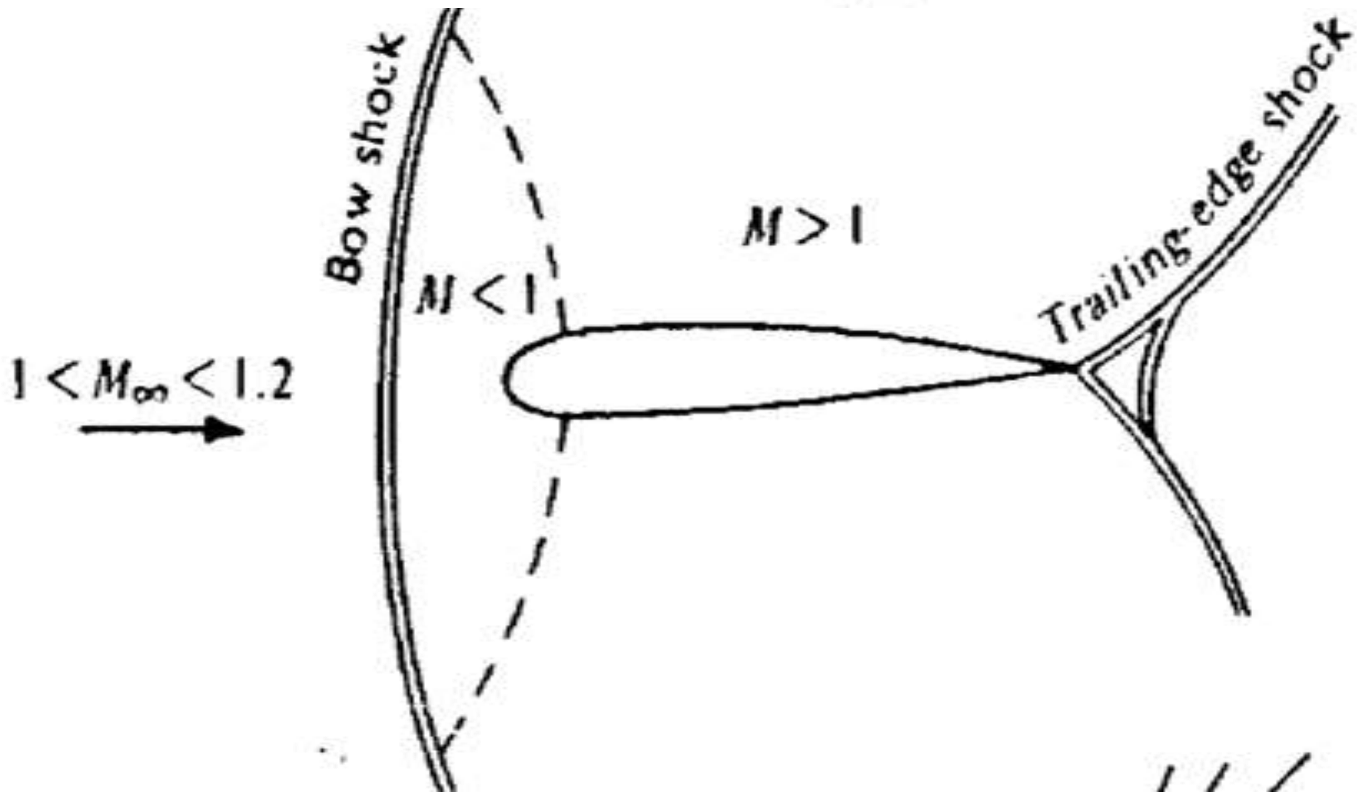


(a) Subsonic flow

Mach Number and the Flight Regime

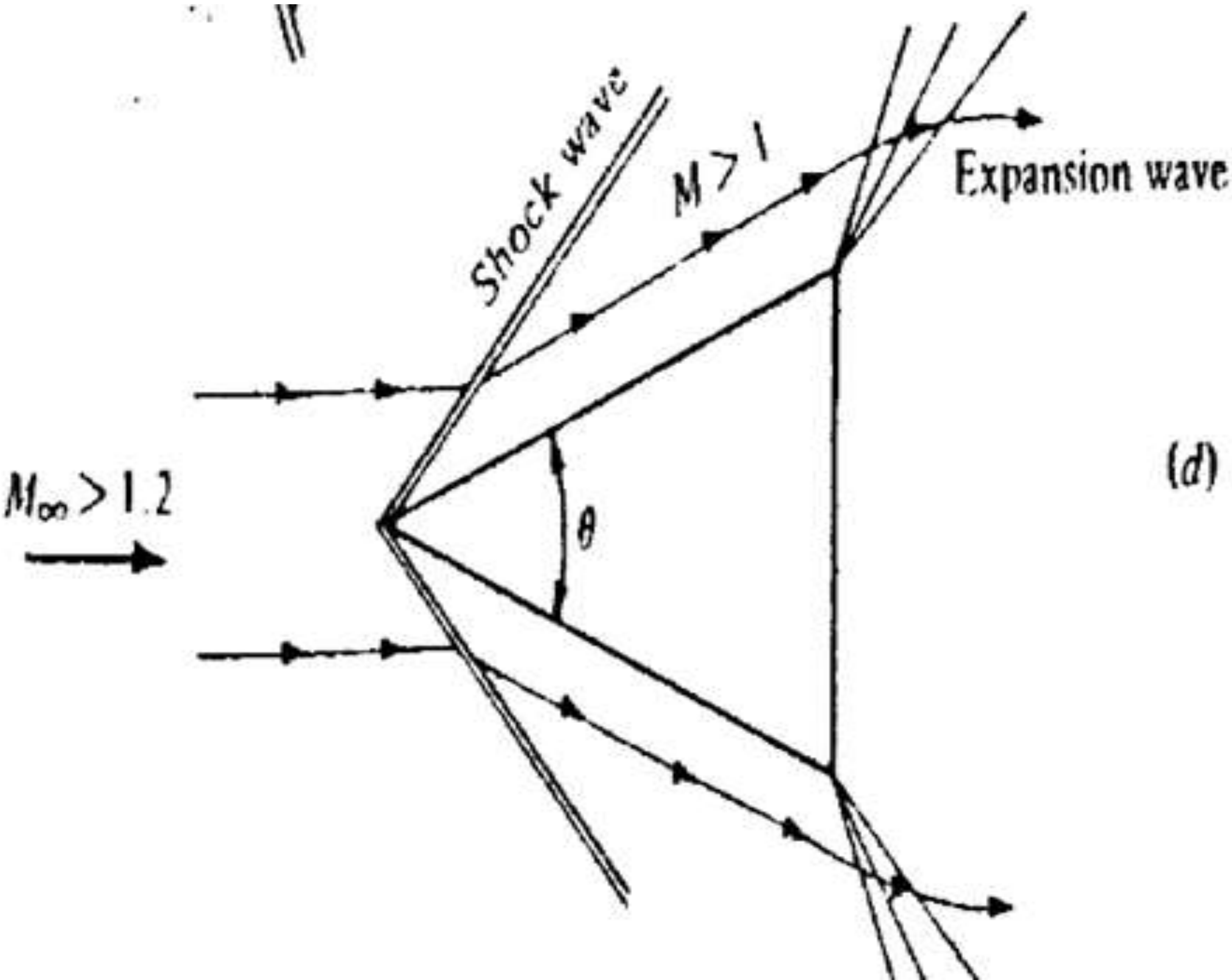


(b) Transonic flow with $M_\infty < 1$



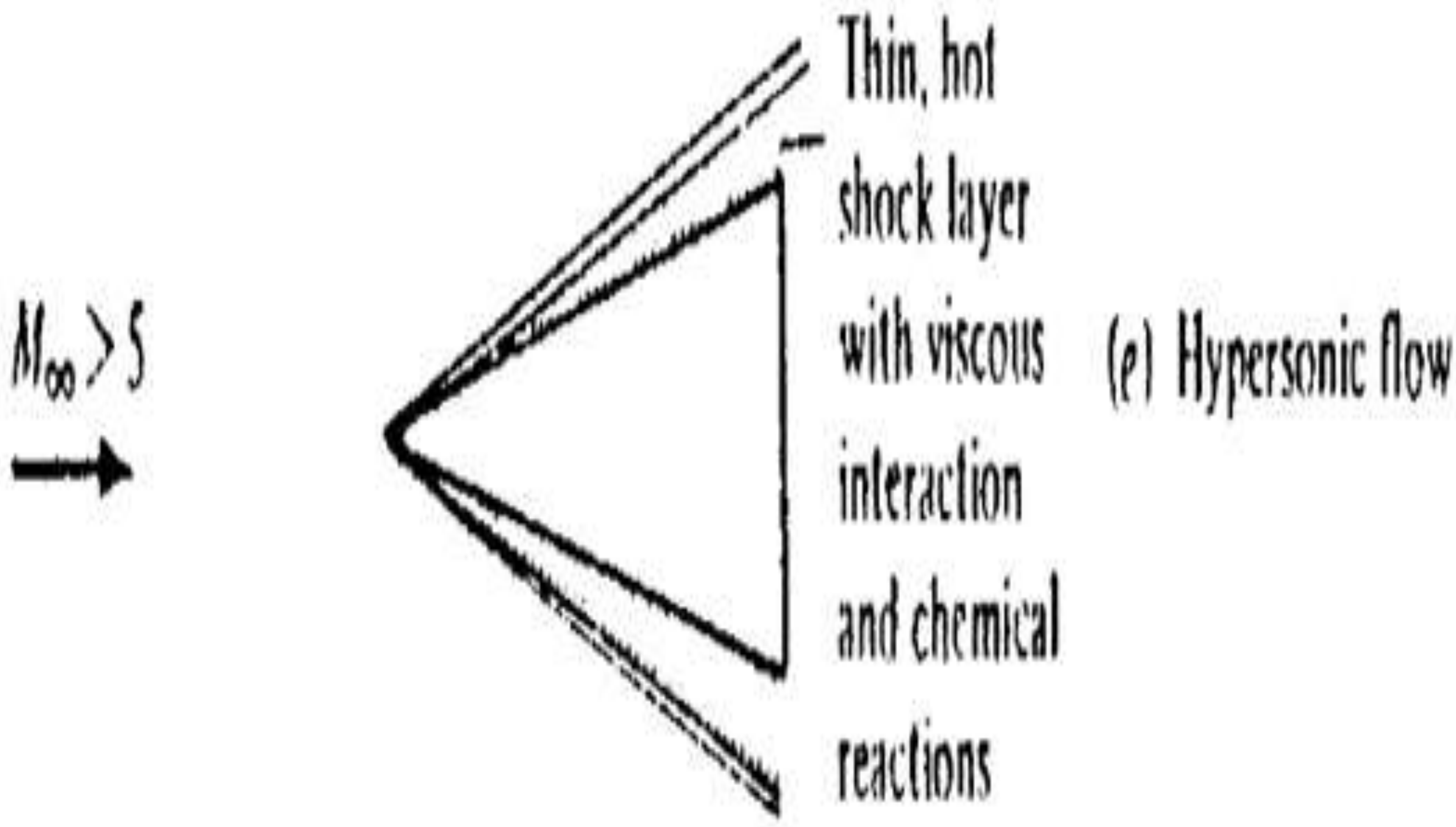
(c) Transonic flow with $M_\infty > 1$

Mach Number and the Flight Regime



(d) Supersonic flow

Mach Number and the Flight Regime



Types of Aircraft in Compressible Flow

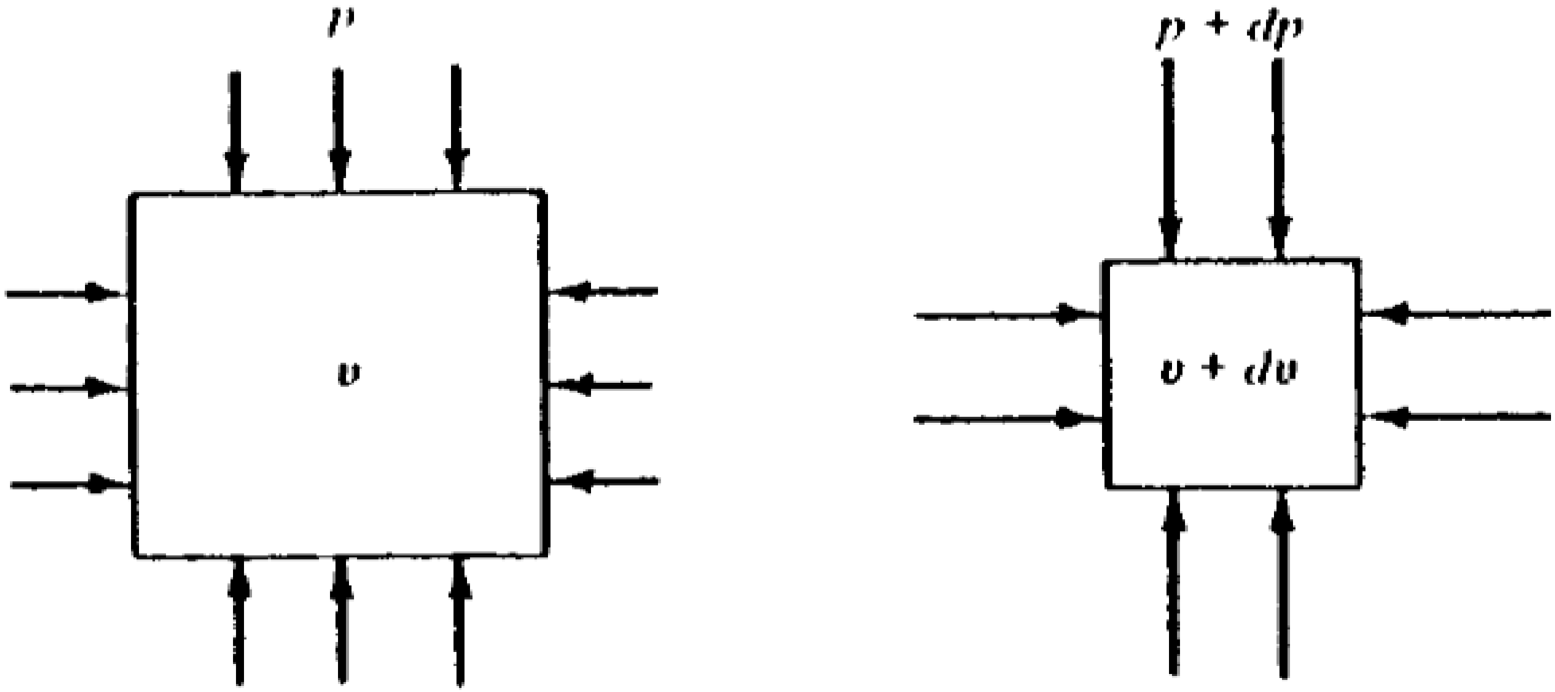
- Almost all general aviation aircraft are subsonic and thus they can be treated as incompressible.
- Military jets and spacecraft on reentry to the atmosphere undergo compressible flow and even hypersonic flow. For example the space shuttle can reach 25 times the speed of sound while on reentry to the atmosphere.

Definition of Compressibility

- It is possible to compress all materials to some extent. The degree which they can be compressed depends on the intrinsic materials of the property.
- Compressibility of a material is the amount of change in the density that will take place when the material is compressed and it is defined by:

$$\tau = -\frac{1}{v}\frac{dv}{dp}$$

Definition of Compressibility



$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

Equations for Inviscid Compressible Flow

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} = 0$$

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y$$

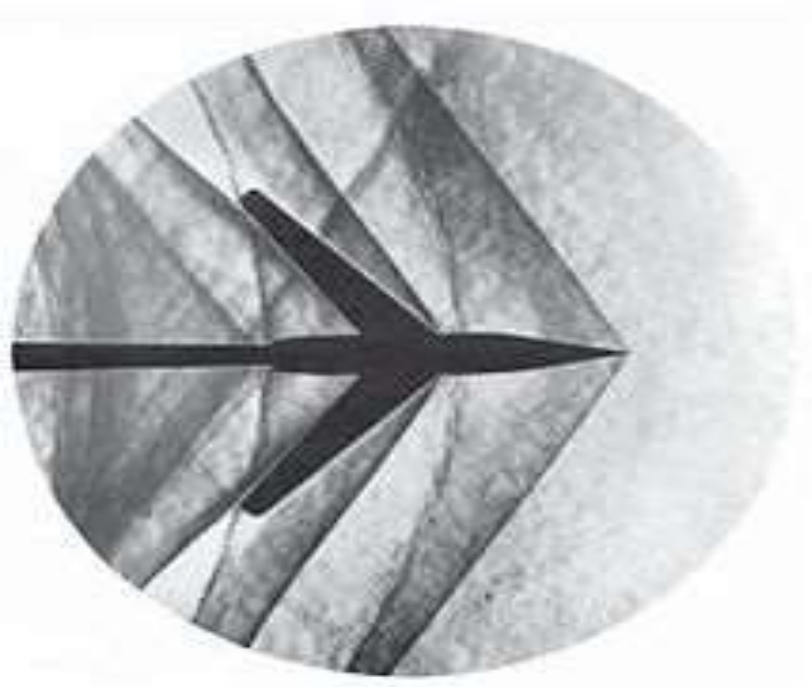
$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z$$

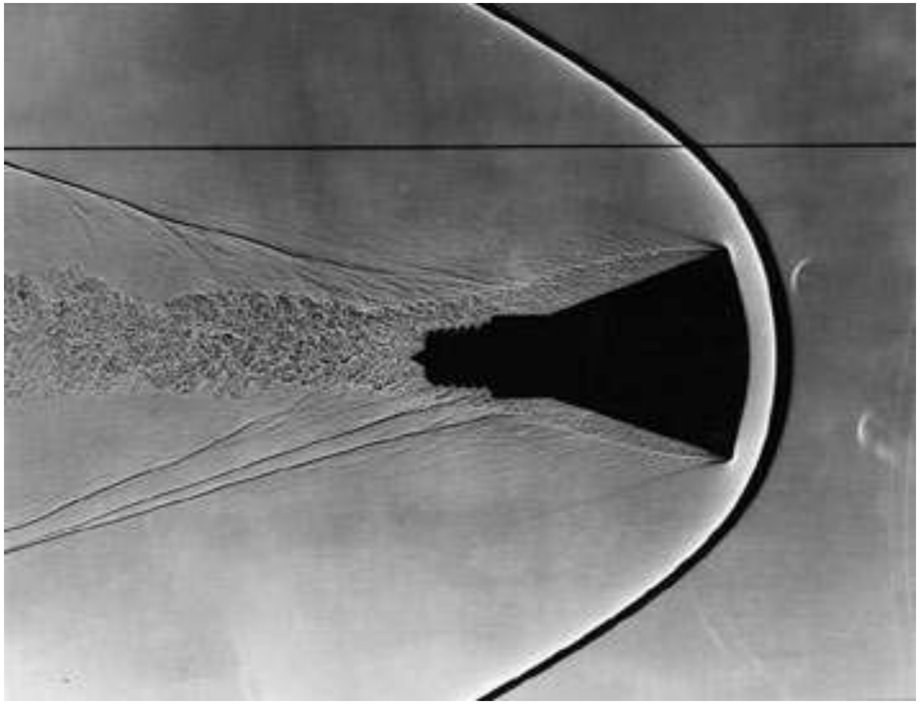
$$\rho \frac{D(e + V^2/2)}{Dt} = \rho \dot{q} - \nabla \cdot p\mathbf{V} + \rho(\mathbf{f} \cdot \mathbf{V})$$

Shockwaves in Compressible Flow

- Whenever there is a flow that is faster than a speed of sound, then shock waves can occur as the aircraft travels in a compressible region.
- A shockwave is an extremely thin region where the flow properties change drastically. They are usually in the order of 0.00001 cm.
- A shockwave is a explosive compression process where the pressure increases discontinuously across the wave.
- This huge level of compression and pressure increase can have adverse effects if the craft is not made supersonic resistant.

Shockwaves in Compressible Flow





Shockwaves

Shockwaves can be Visible by Condensation

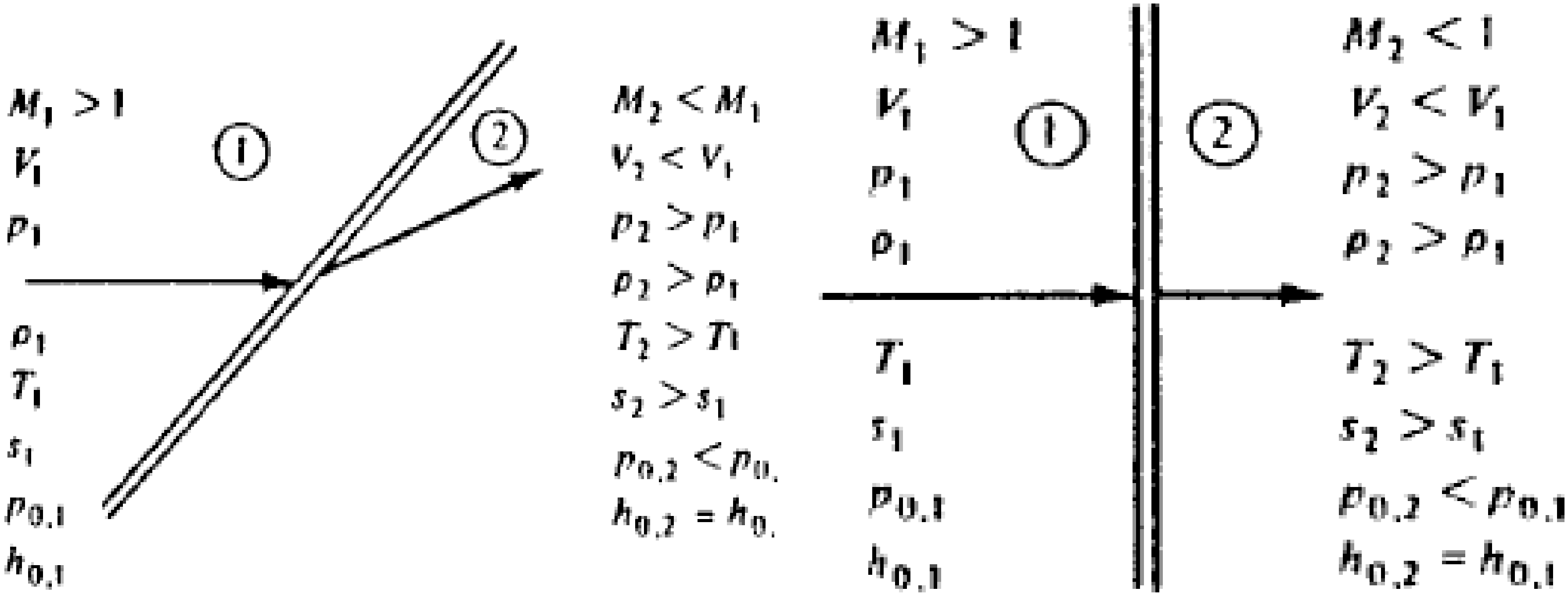
Characteristics of Shock Waves

- The biggest characteristics of a shockwave is that the pressure will increase greatly as the shockwave is passed.
- When a shock wave passes through matter, the total energy is preserved but the energy which can be extracted as work decreases and entropy increases. This, for example, creates additional drag force on aircraft with shocks

General Properties of Shockwaves

- When you pass across a shockwave:
- Pressure increases $P2 > P1$
- Temperature increases $T2 > T1$
- Density increases
- Entropy increases
- Mach number decreases $M2 < M1$
- Flow velocity decreases

General Properties of Shockwaves



(*a*) Oblique shock wave

(*b*) Normal shock wave

Thermodynamic Properties of Compressible Flow

- Compressible flow and shockwaves are actually related to thermodynamics, as heat exchange takes place. Here are some important general thermodynamic equations to remember:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

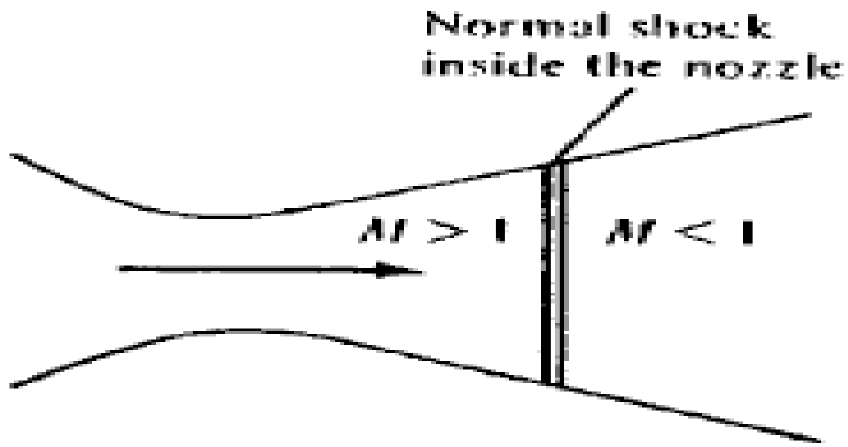
$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

ISENTROPIC Process
(Reversible Adiabatic Flow)

This applies to all compressible flows outside the Boundary Layer Region

Shock Waves in Nozzles

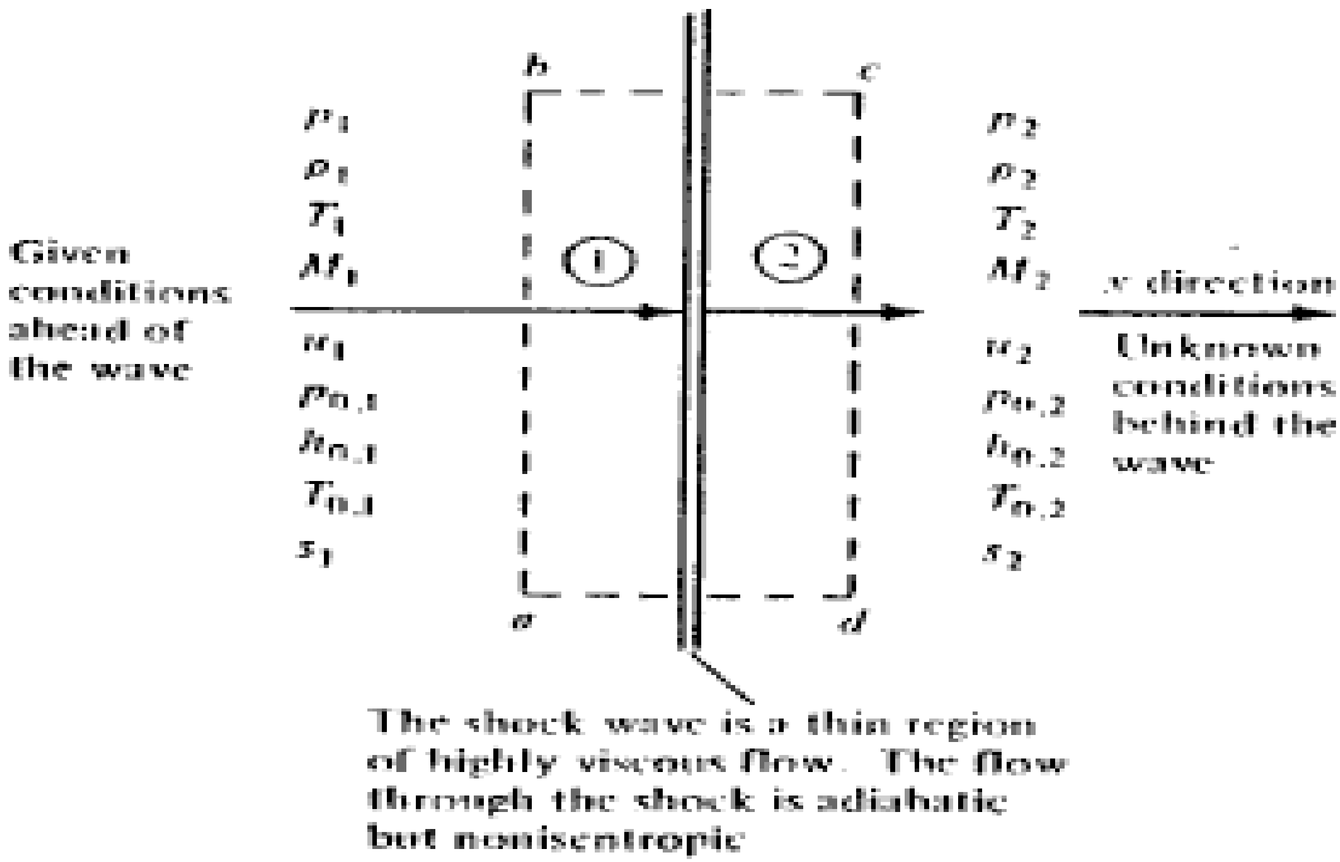
- Especially in rockets, you will have a main nozzle where the fuel of the rocket will burn and be discharged. This is called a convergent – divergent nozzle. While the flow is expanding in the nozzle, there will be shock waves due to high speeds and compressibility involved.



Overexpanded flow
through a nozzle

Conditions of a Shockwave

1) The flow is considered steady over time.
2) The flow is adiabatic in the sense that no heat is added to the control volume. However, temperature rises across the shock wave even though no heat is added. Why?
3) No Viscous effects outside the boundary layer.
4) No body forces f=0



Equations for a Shock Wave

- The basic equations for a normal shock wave:

Continuity: $$\rho_1 u_1 = \rho_2 u_2$$

Momentum: $$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

Energy: $$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

Enthalpy: $$h_2 = c_p T_2$$

Equation of state: $$p_2 = \rho_2 R T_2$$

Calculation of Normal shock wave Properties

- There are several equations that are in use for calculating normal shock waves. However, the most important result is the fact that the Mach number M2 behind the wave is a function of Mach number ahead of the wave. Also if M1>1 then M2<1 as the Mach number behind the wave becomes subsonic.

$$M_2^2 = \frac{1 + [(\gamma - 1)/2]M_1^2}{\gamma M_1^2 - (\gamma - 1)/2}$$

Calculation of Normal shock wave Properties

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma + 1)M_1^2}{2 + (\gamma - 1)M_1^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)\right]\frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}$$

As it can be seen, the pressure, the density and the temperature on the other side of the shockwave is directly the property of M1. Hence, the upstream Mach number determines the properties of compressible flow.

Why Does Entropy Increase Across the Shockwave?

Why does the entropy increase across the shock wave? The second law tells us that it must, but what mechanism does nature use to accomplish this increase? To answer these questions, recall that a shock wave is a very thin region (on the order of 10^{-5} cm) across which some large changes occur almost discontinuously. Therefore, within the shock wave itself, large gradients in velocity and temperature occur; i.e., the mechanisms of friction and thermal conduction are strong. These are dissipative, irreversible mechanisms that always increase the entropy. Therefore, the precise entropy increase predicted by Eq. (8.68) for a given supersonic M_1 is appropriately provided by nature in the form of friction and thermal conduction within the interior of the shock wave itself.

Characteristic Mach Number

In the definition of M, a is the local speed of sound, $a = \sqrt{\gamma RT}$. In the theory of supersonic flow, it is sometimes convenient to introduce a "characteristic" Mach number, M^*, defined as

$$M^* \equiv \frac{u}{a^*}$$

where a^* is the value of the speed of sound at sonic conditions, *not* the actual local

$$M^{*2} = \frac{(\gamma + 1)M^2}{2 + (\gamma - 1)M^2}$$