# Aerodynamics Lecture Notes

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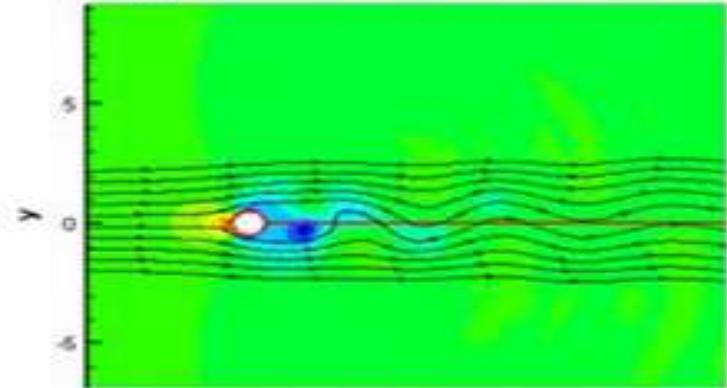**Nuclear Spacecraft Propulsion Specialist**

Viscous Flow

- In reality every flow in the world is a viscous flow.

- Viscosity is the phenomena of friction that acts on all objects and fluids.

- In viscous flow, friction of surfaces, heat transfer and energy transfer between molecules and mass transfer (diffusion) takes place.
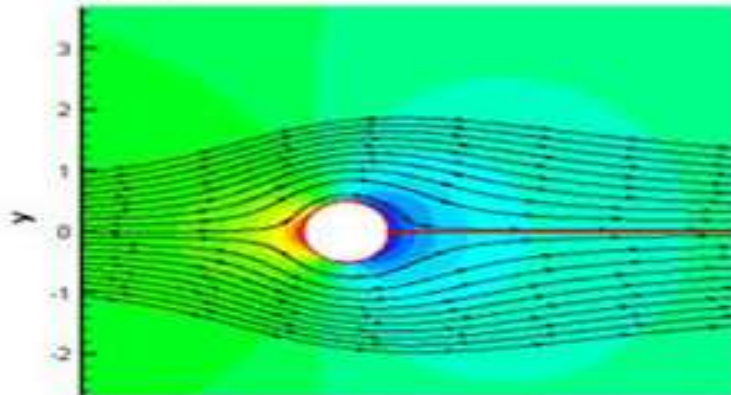
Examples of Viscous Flow
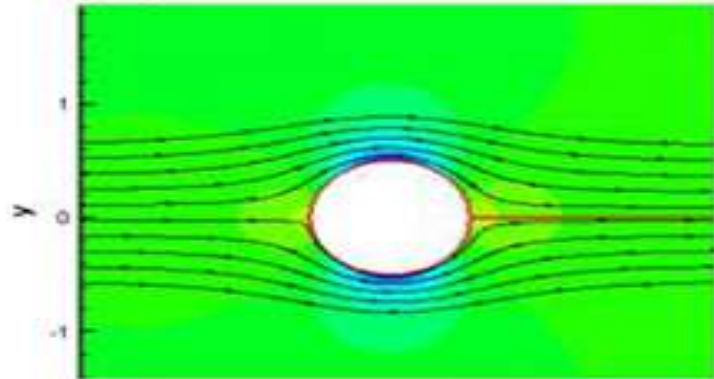
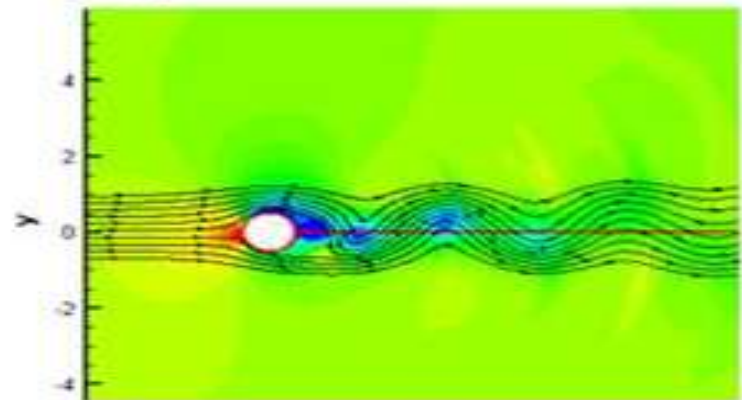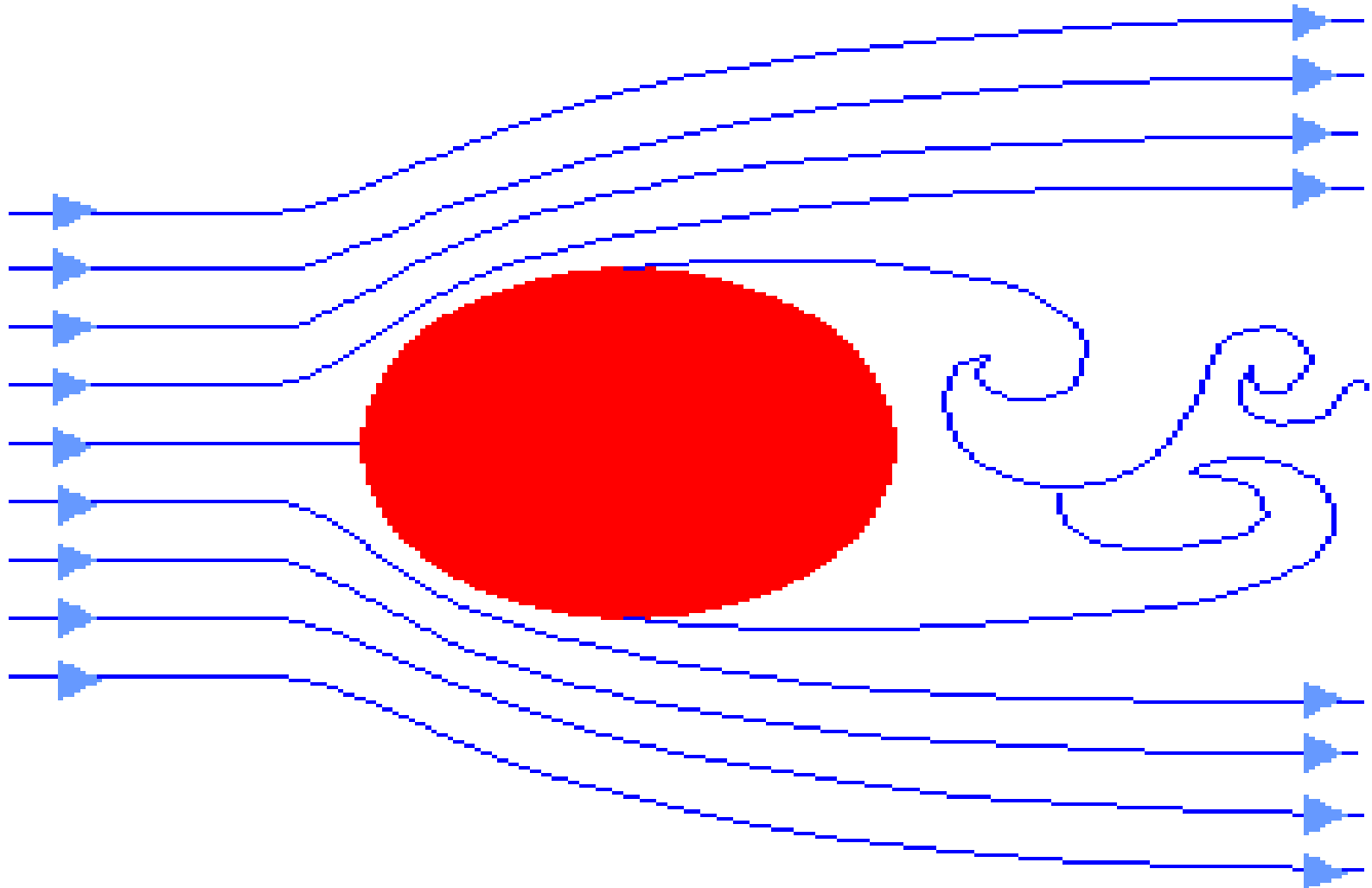# Examples of Viscous Flow

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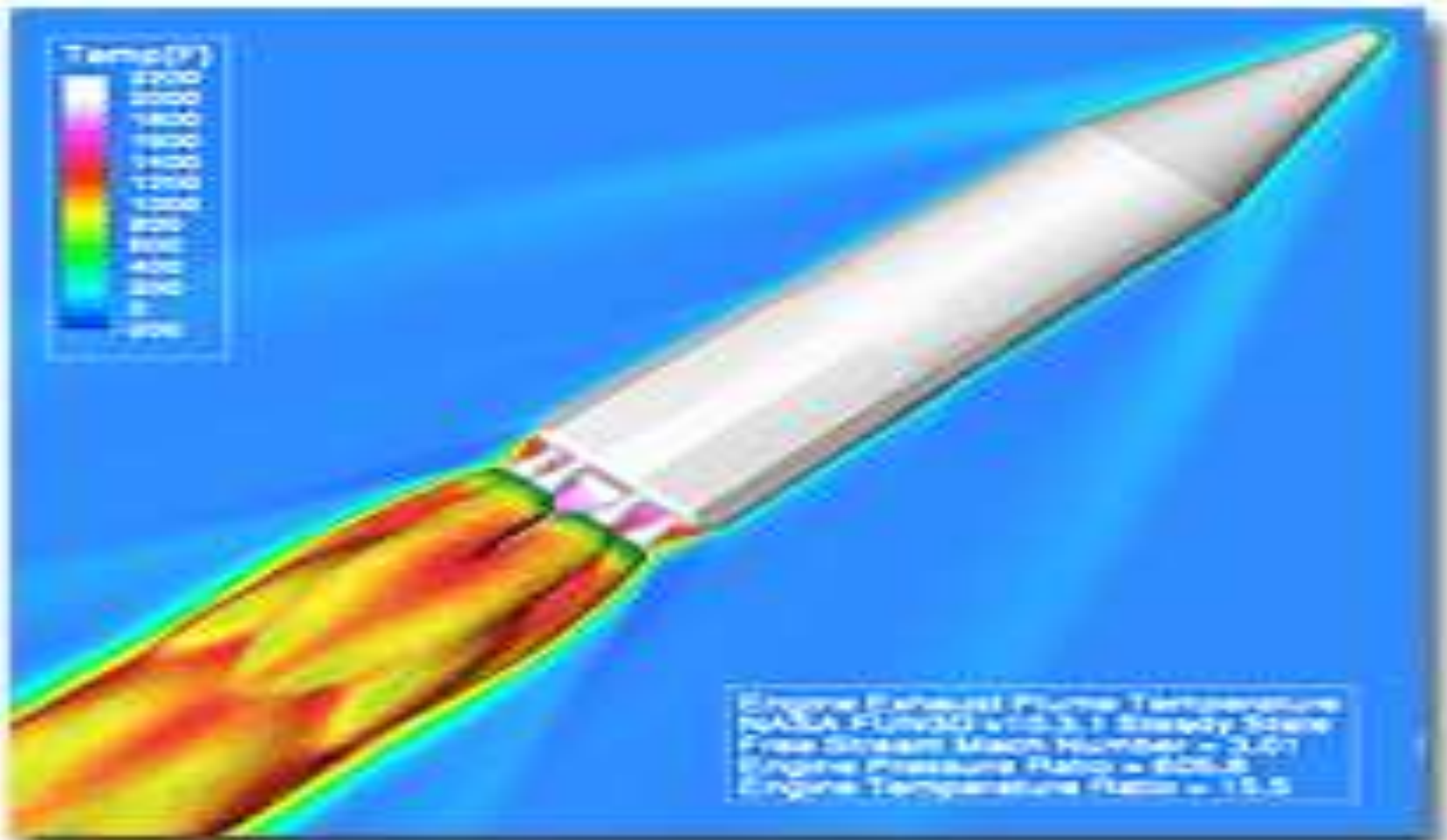
Examples of Viscous Flow
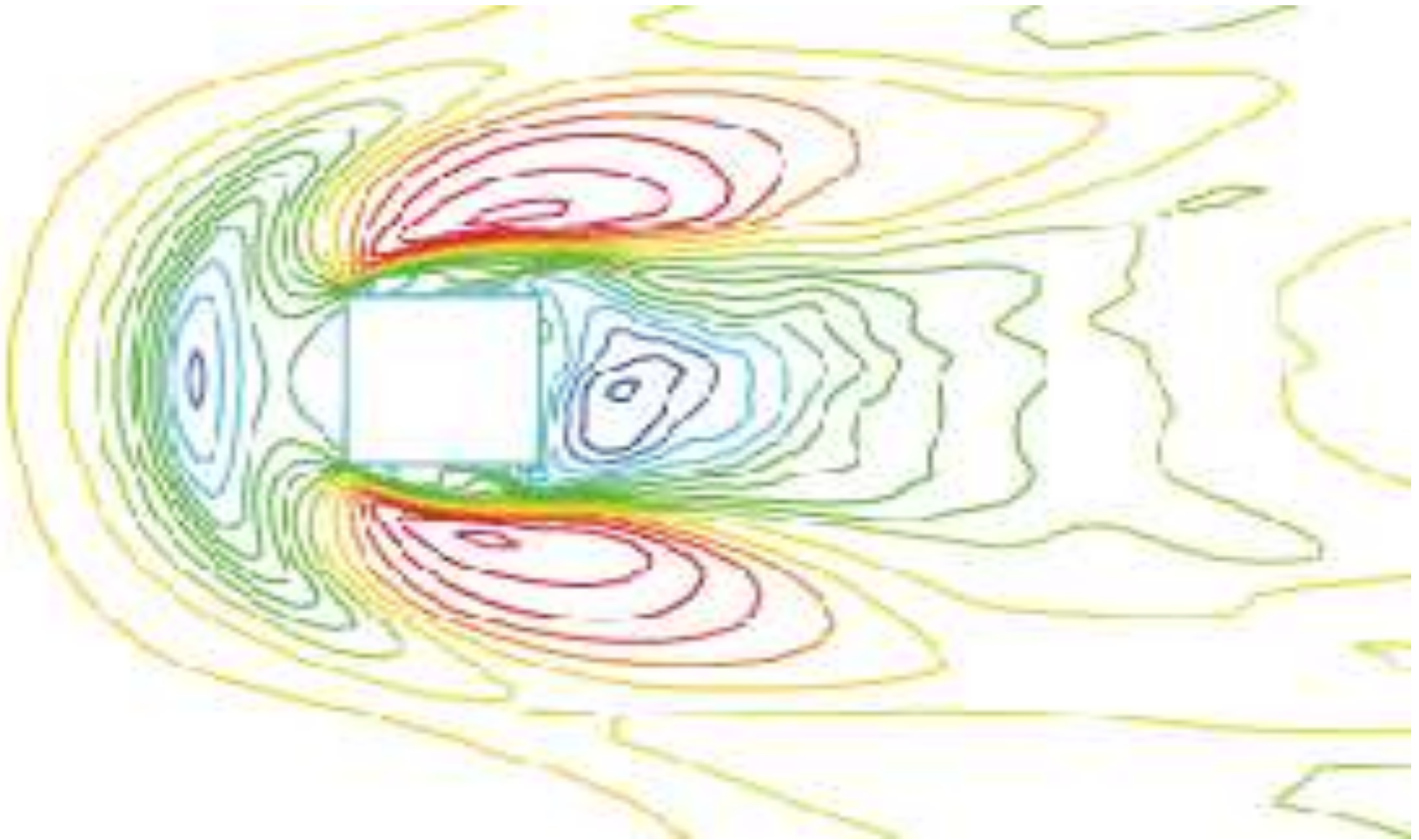
Friction (Viscosity) Creates Shear Stress
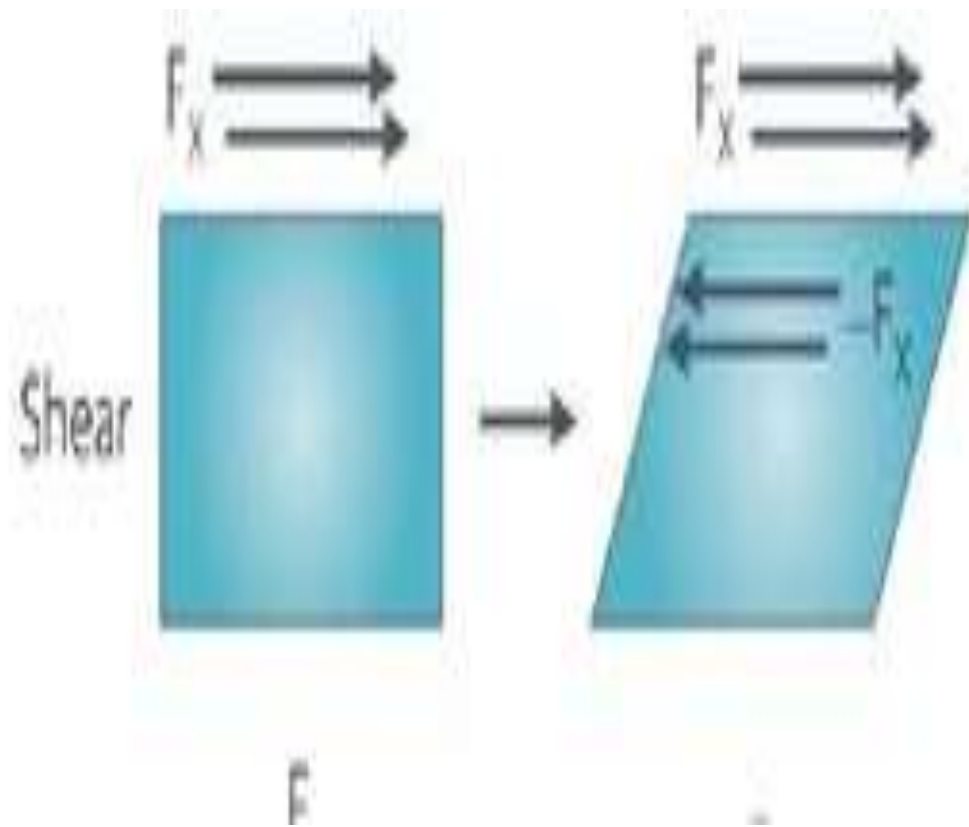
Shear stress is stress applied parallel to the surface of the cell.

No Slip Condition

- The influence of friction creates V=0 at the body surface and this is called no-slip condition.

fluid

solid

n

d_b d d_a a

b
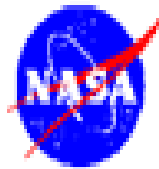
Boundary Layer

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Laminar

Velocity

Free Stream

Turbulent

Velocity

Boundary Layer

Unsteady

Surface of Object
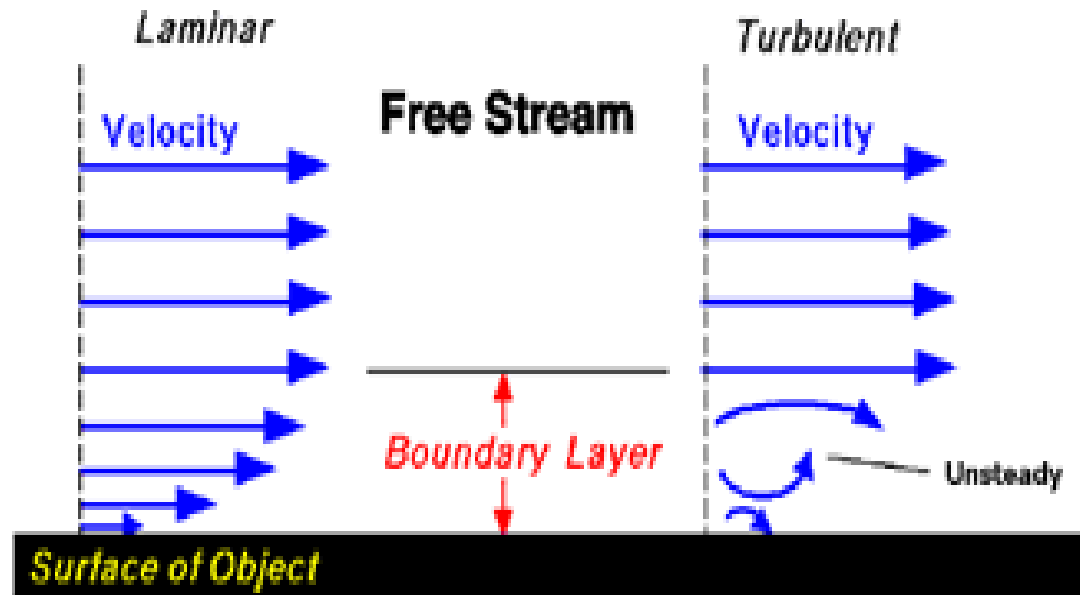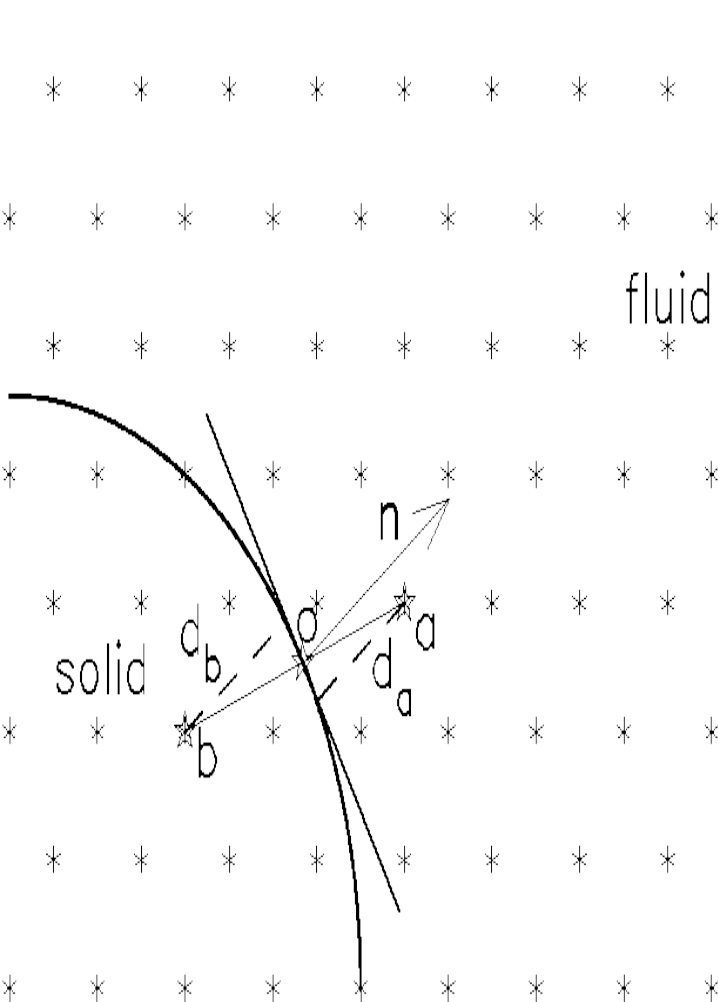
Velocity is zero at the surface (no-slip)

Adverse Pressure Gradients

- If the flow on the surface produces an increasing pressure distribution in the flow direction (P3 > P2 > P1) then such a region is called an adverse pressure gradient.

- This can create **REVERSED FLOW** in the field
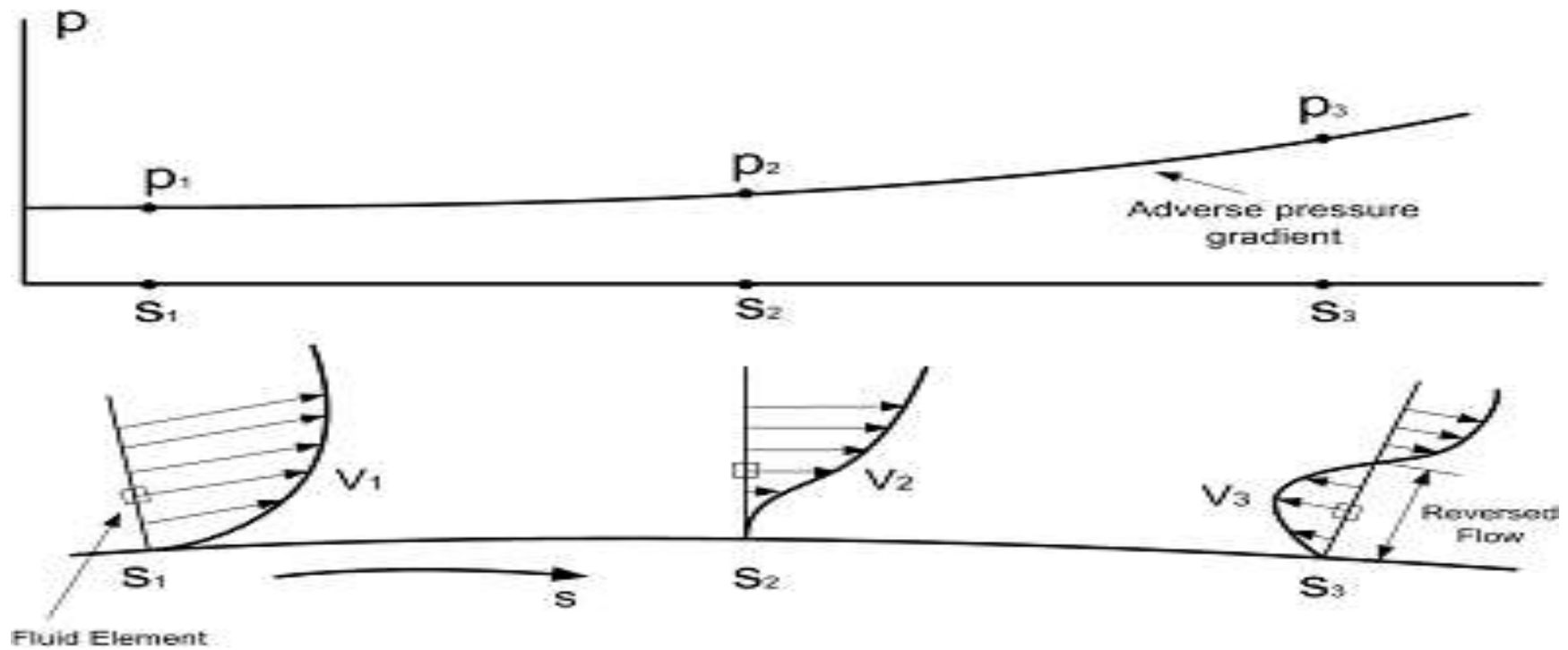
Reversed Flow

- When the fluid element is retarded by friction and increasing pressure, it will slow down and eventually stop as it travels downstream. Then, as pressure mounts, it will reverse direction and move back upstream (Reversed Flow)
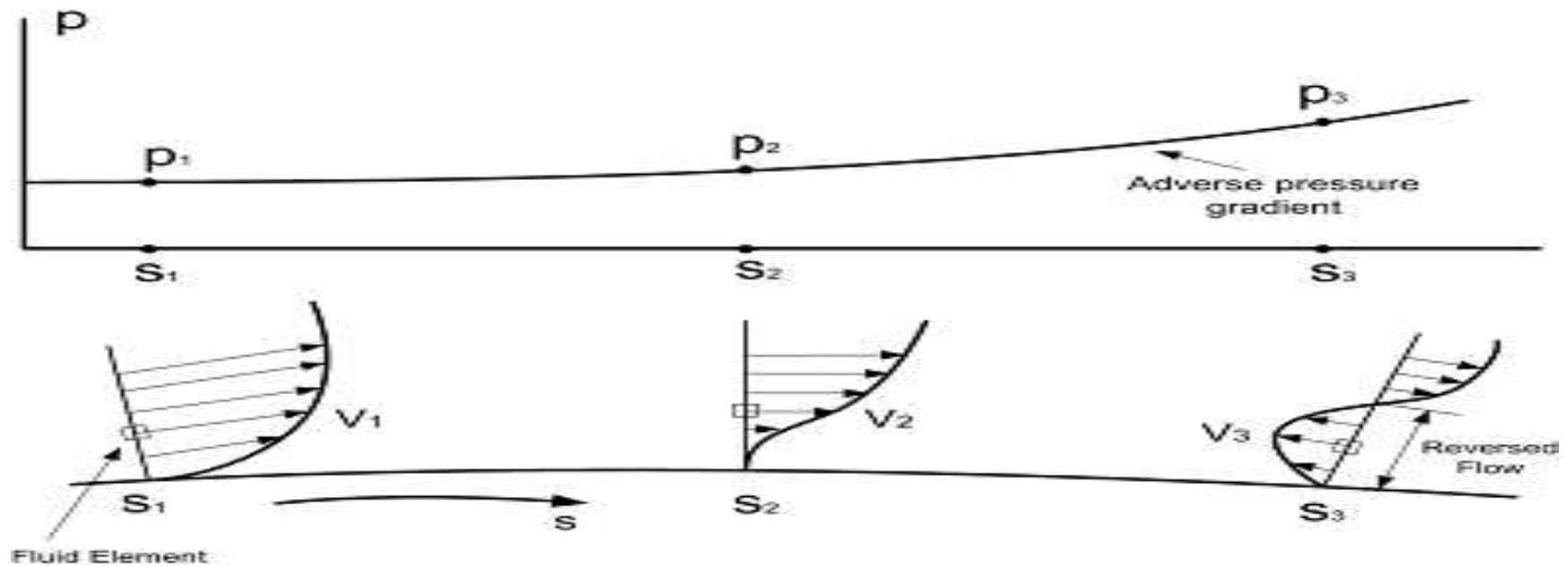
Flow Separation

- Reversed flow will cause the flow to separate from the surface and create a large wake of recirculating flow downstream of the surface.

- The point of separation occurs at dV/dN = 0

Relative drag force

(a) Blunt body

(b) Streamlined body

Separated flow

Separated flow

Code
■ Skin friction drag
□ Pressure drag
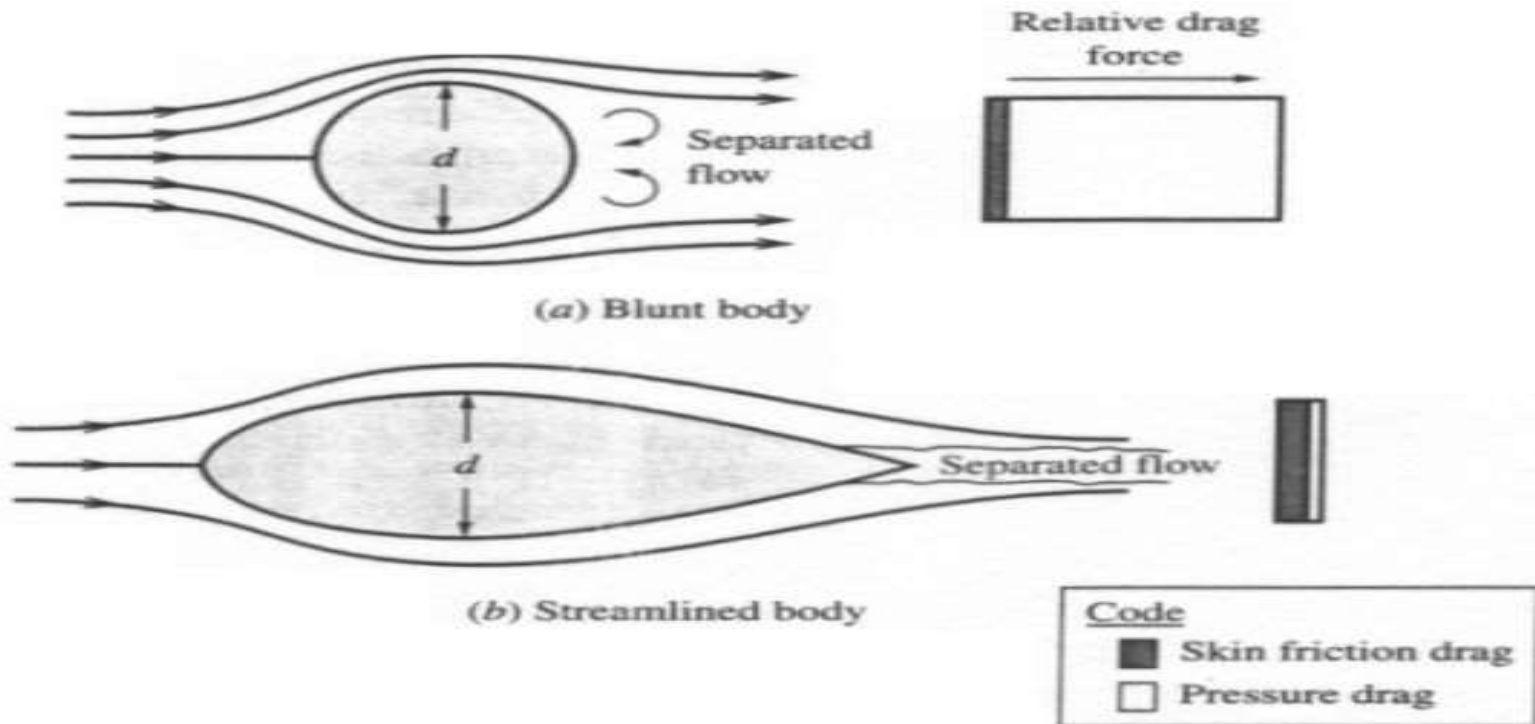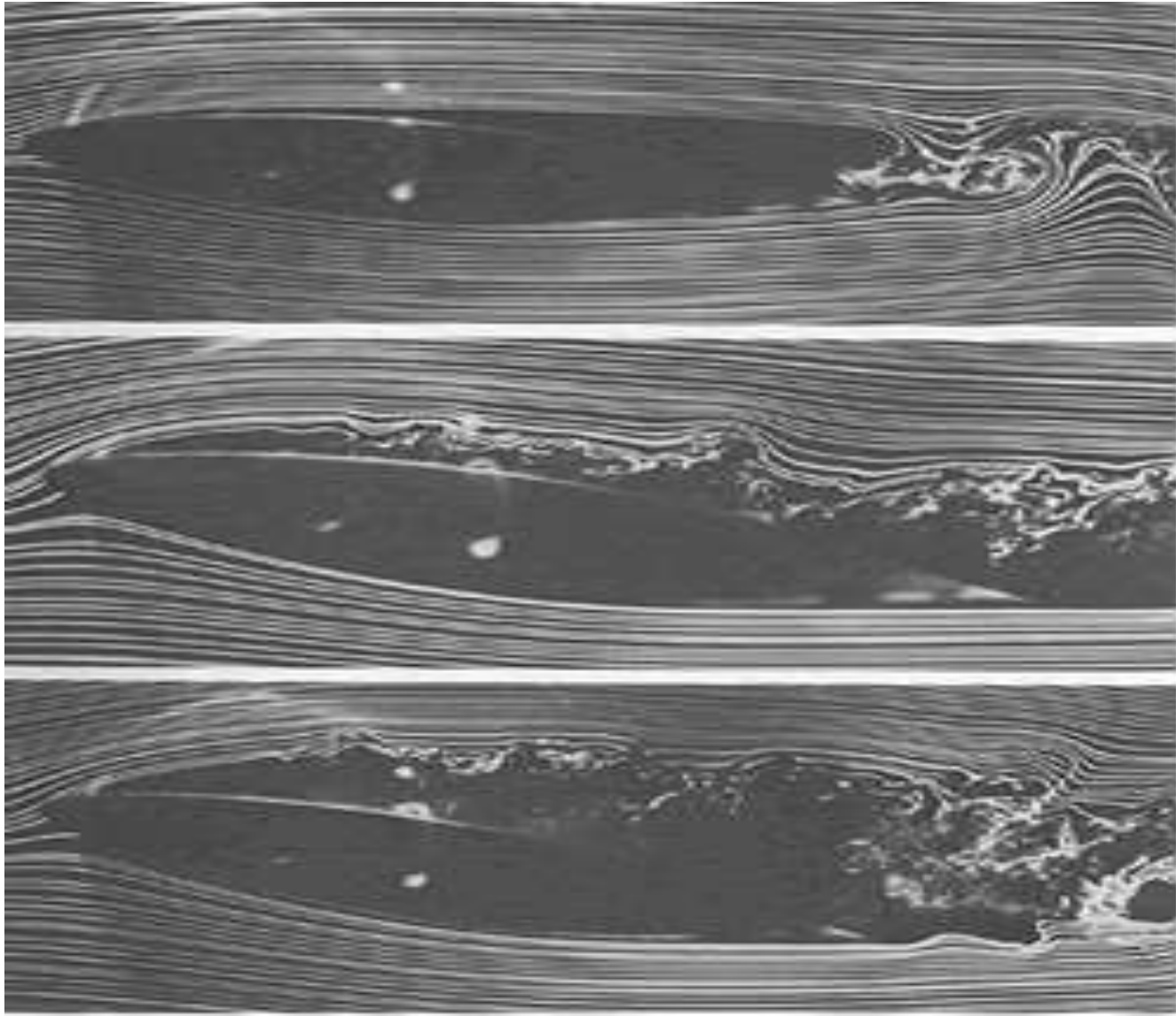
Flow Separation

Flow Separation & Airfoil Stall

- Separation of flow increases the drag and results in substantial loss of lift.

- Thus analysis of viscous flow and flow separation is important in order to prevent airfoil stall.

Flight direction

Geometric Angle of Attack
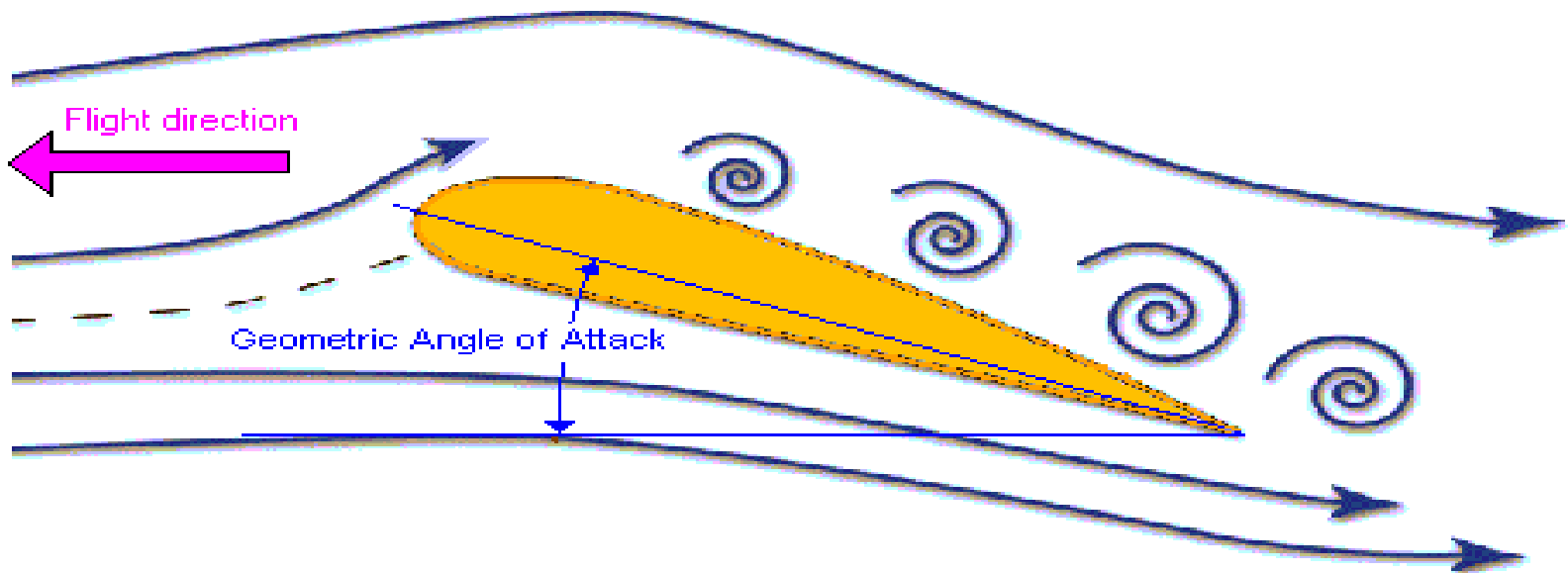
Flow Separation & Airfoil Stall

Inclination Effects on Drag

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Chord

Stalled

Flight Path

a

Attached

Drag

Stall

Angle of Attack = a
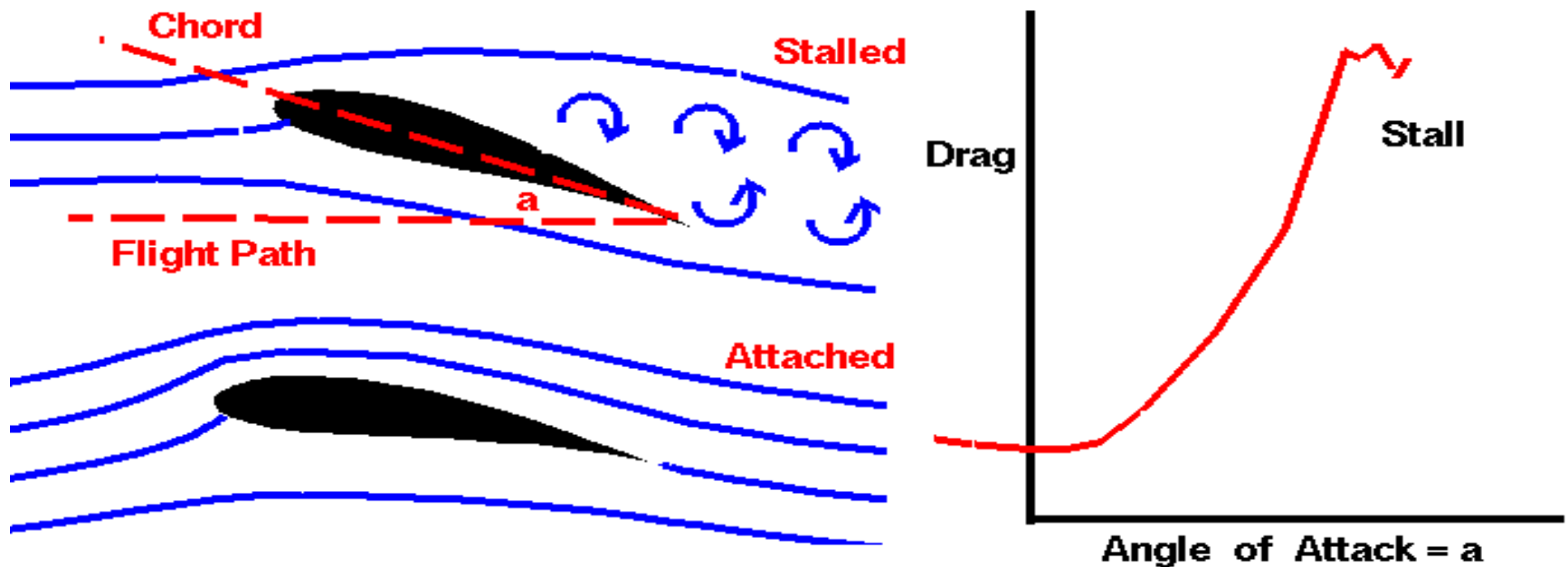
For small angles, drag is nearly constant.
As angle increases, drag increases.
Greater Angle = Greater Drag
After stall, relationship is quite complex.
Included in Drag Coefficient

Effects of Viscosity on Shear Stress

- Hence, as it can be seen from the slides above, viscosity can cause reversed flow as well as flow separation which can effect the stability of the airfoil.
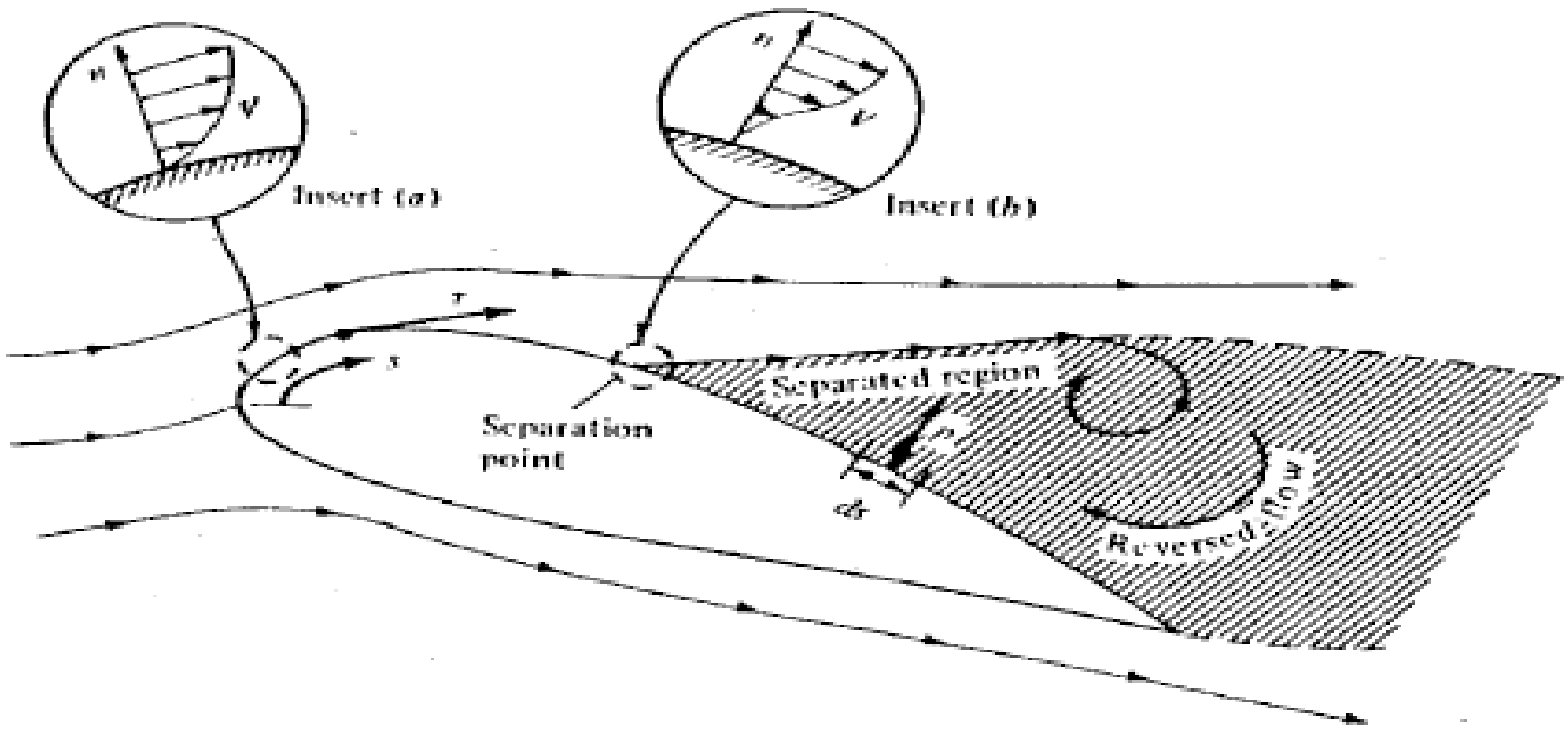
Types of Drag Due to Viscous Flow

- There are two main types of drag on the airfoil surface due to Viscous flow effects.

- These are D_f which is Skin friction drag

- D_p which is the pressure drag of separation

- These numbers will effect whether we want turbulent or laminar flow for better control of the airfoil surface

$$D = D_f + D_p$$

Heat Transfer in Viscous Flow

- The moving fluid has a certain amount of energy. As it flows on the surface, the flow velocity is decreased due to friction.

- This decrease in velocity is translated as a decrease in kinetic energy.

- This loss of kinetic energy transforms into internal energy of the fluid. (Conservation of Energy)

- Hence, the temperature of the fluid will rise.

Aerodynamic Heating in Viscous Flow

- As a result, warmer fluid will heat the cooler surface of the body.

- This is called **aerodynamic heating**. This becomes more severe since as flow velocity increases aerodynamic heating will also increase.
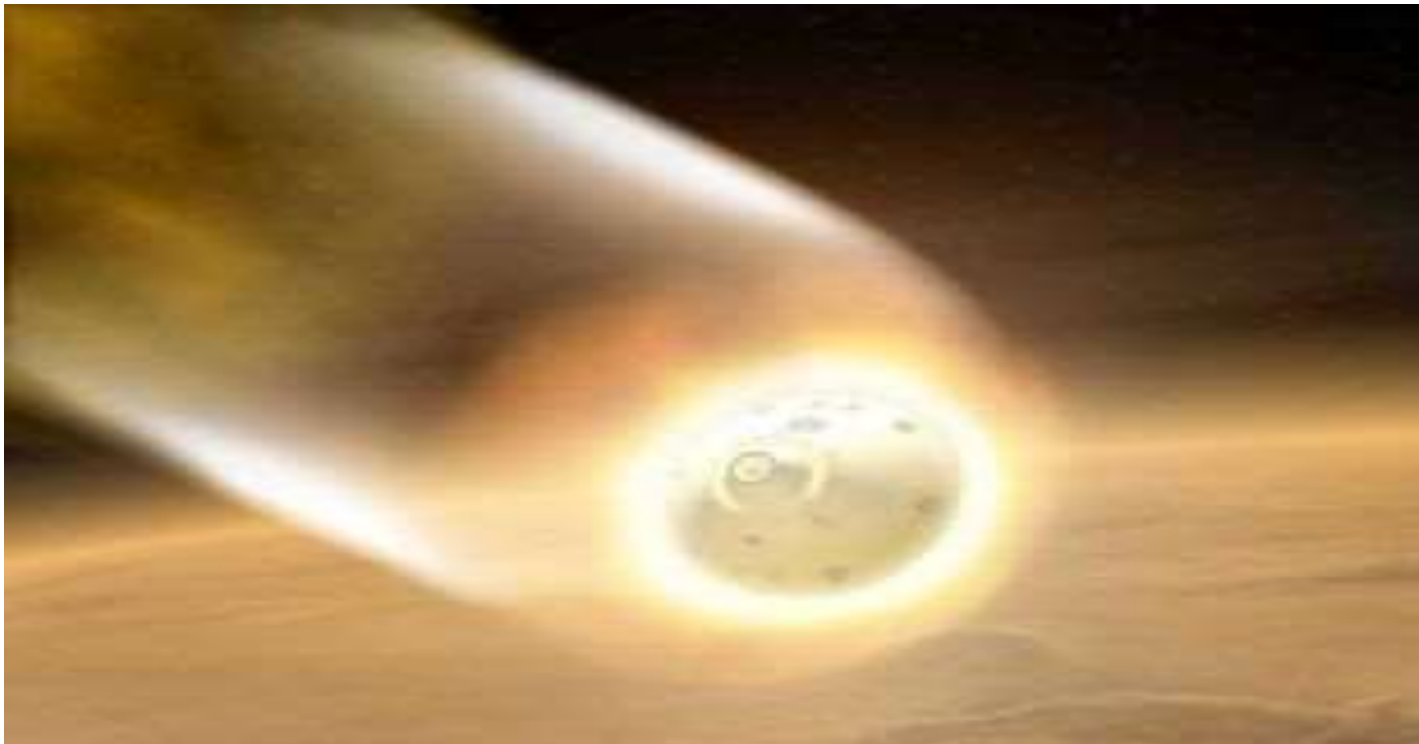
Laminar and Turbulent Flow

- It is very important to understand the effects of viscous flow on laminar and turbulent flow
- If the flow is smooth, it is called **laminar flow**. If the flow is irregular, random and jagged, it is **turbulent flow**.

$$\left[\left(\frac{\partial V}{\partial n}\right)_{n=0}\right]_{turbulent} > \left[\left(\frac{\partial V}{\partial n}\right)_{n=0}\right]_{laminar}$$

Turbulent Viscous Flow

- In turbulent flow, the frictional effects are more severe.

- Also, in turbulent flow, shear stress and aerodynamic heating are greater.
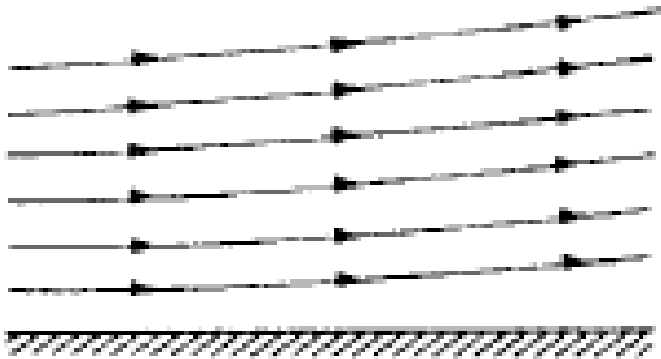
(a) Laminar flow   (b) Turbulent flow

Laminar and Turbulent Flow
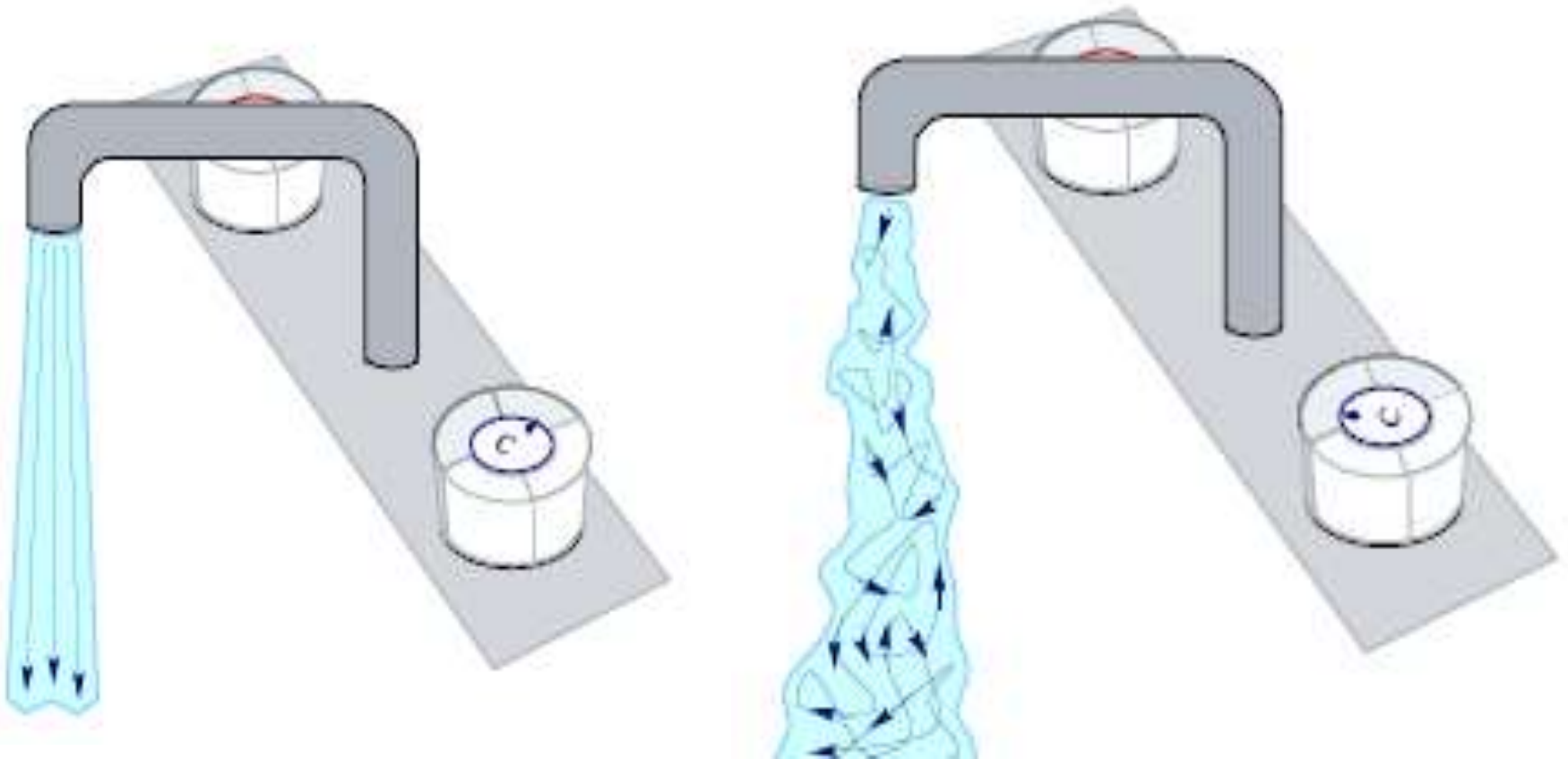
Turbulent Viscous Flow

- However for turbulent flow, there is an advantage as the flow separation is less likely to happen. Even if flow separation occurs, the separation region will be much slower.

- The pressure drag Dp is smaller in turbulent flow.
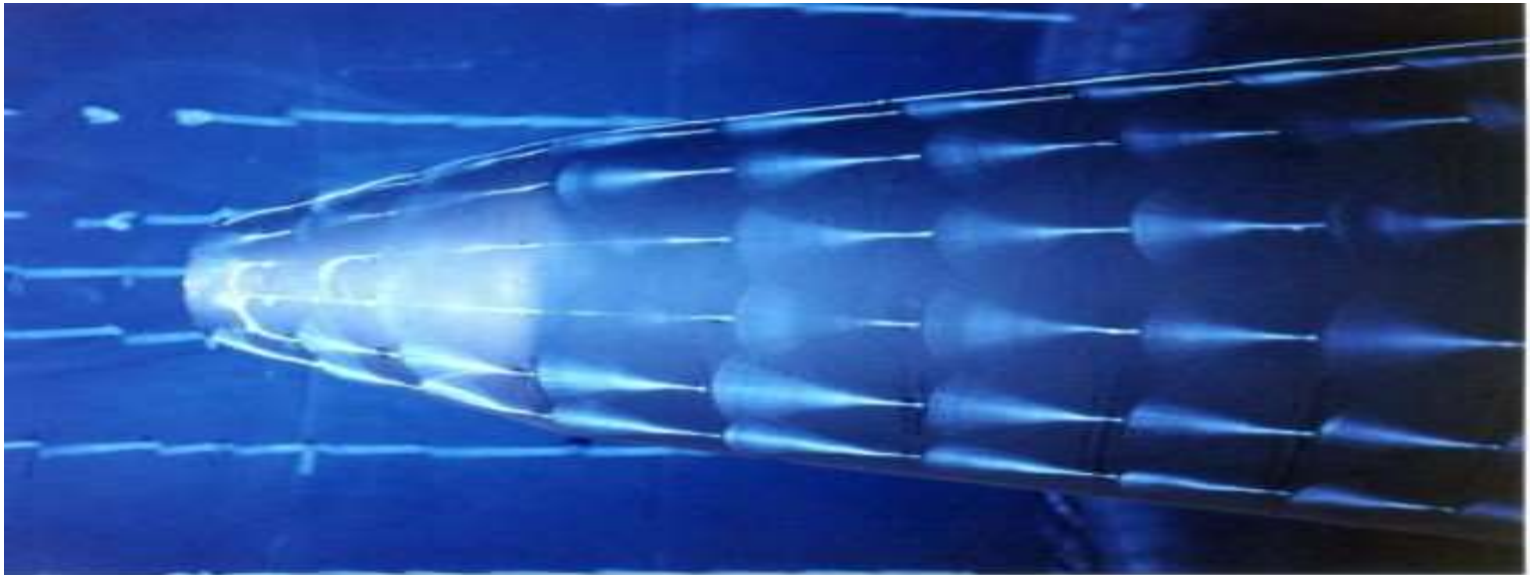
Laminar and Turbulent Flow

- There are certain conditions which cause laminar flow to become transitional and turn into turbulent flow. These reasons include:

  a) Increased Surface Roughness

  b) Increased turbulence in the free stream

  c) Adverse pressure gradients

  d) Heating of the fluid by the surface will cause turbulent flow

Turbulent
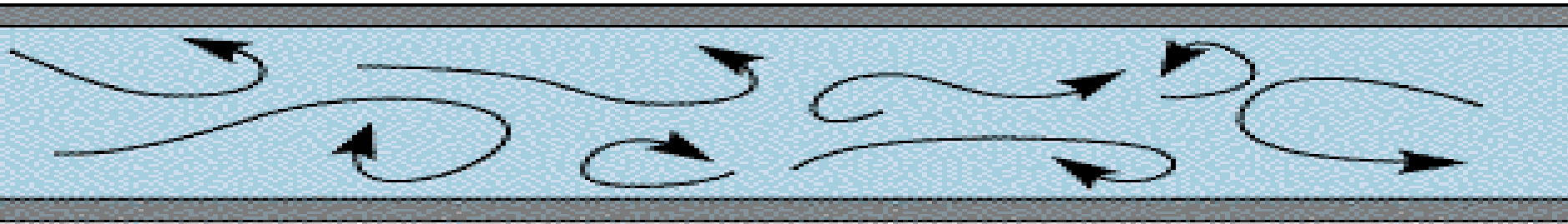
Laminar
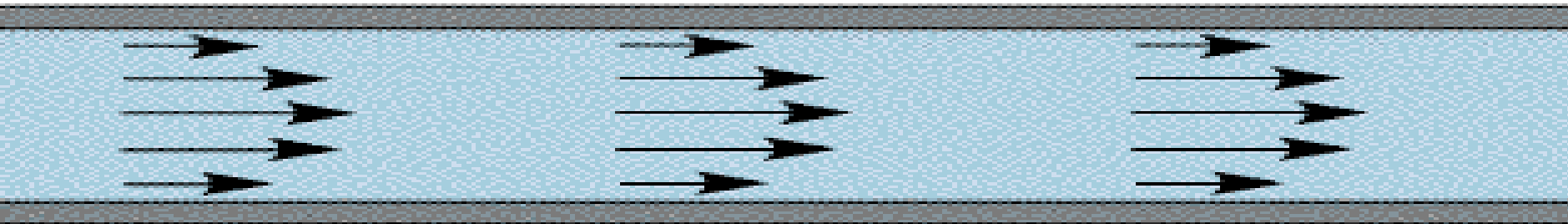
Transition to Turbulent Flow

laminar — transitional — turbulent
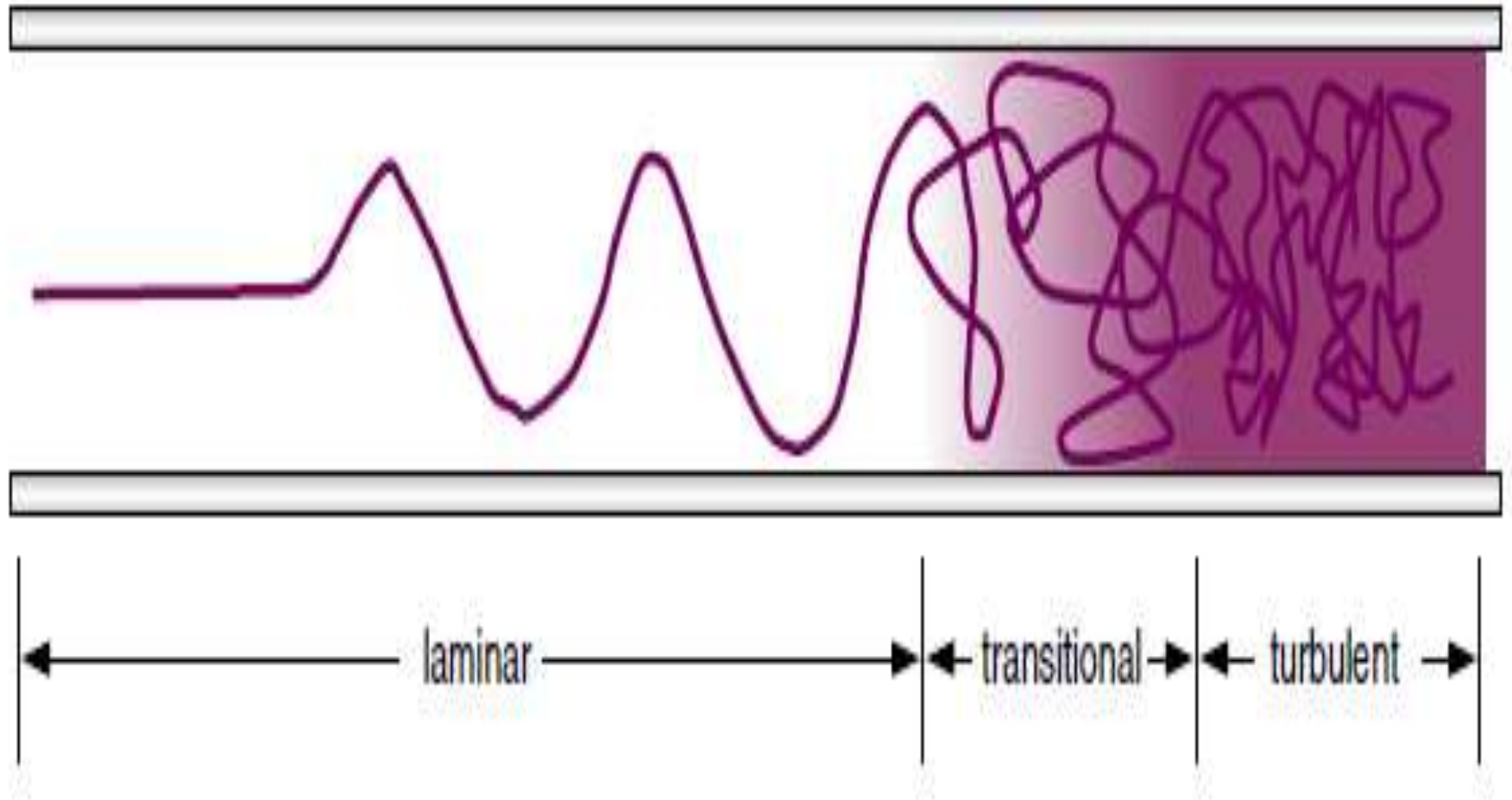
When is Turbulent Flow Desirable?

- There are some cases where turbulent flow may be desirable instead of laminar flow.

- For example for slender bodies, laminar flow is desirable, while for blunt bodies turbulent flow may be desirable.

- Usually Re<< 500,000 = Laminar Flow

- Re>>500,000 = Turbulent Flow

$$D_f \gg D_p$$
Laminar flow desirable

(a) Slender body

$$D_p \gg D_f$$
Turbulent flow desirable

(b) Blunt body
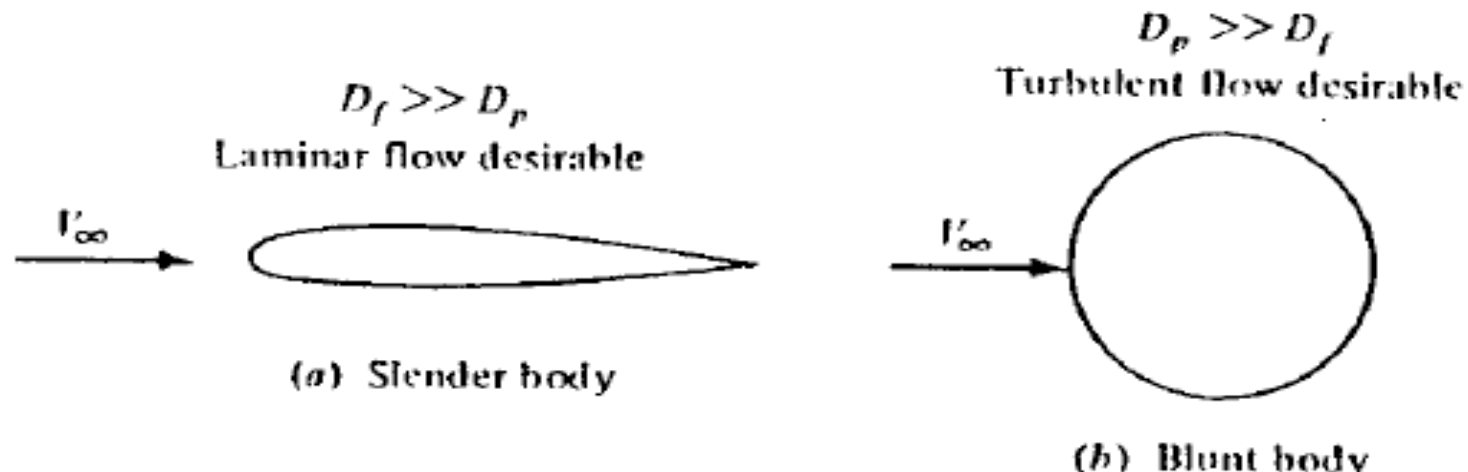
When is Turbulent Flow Desirable?
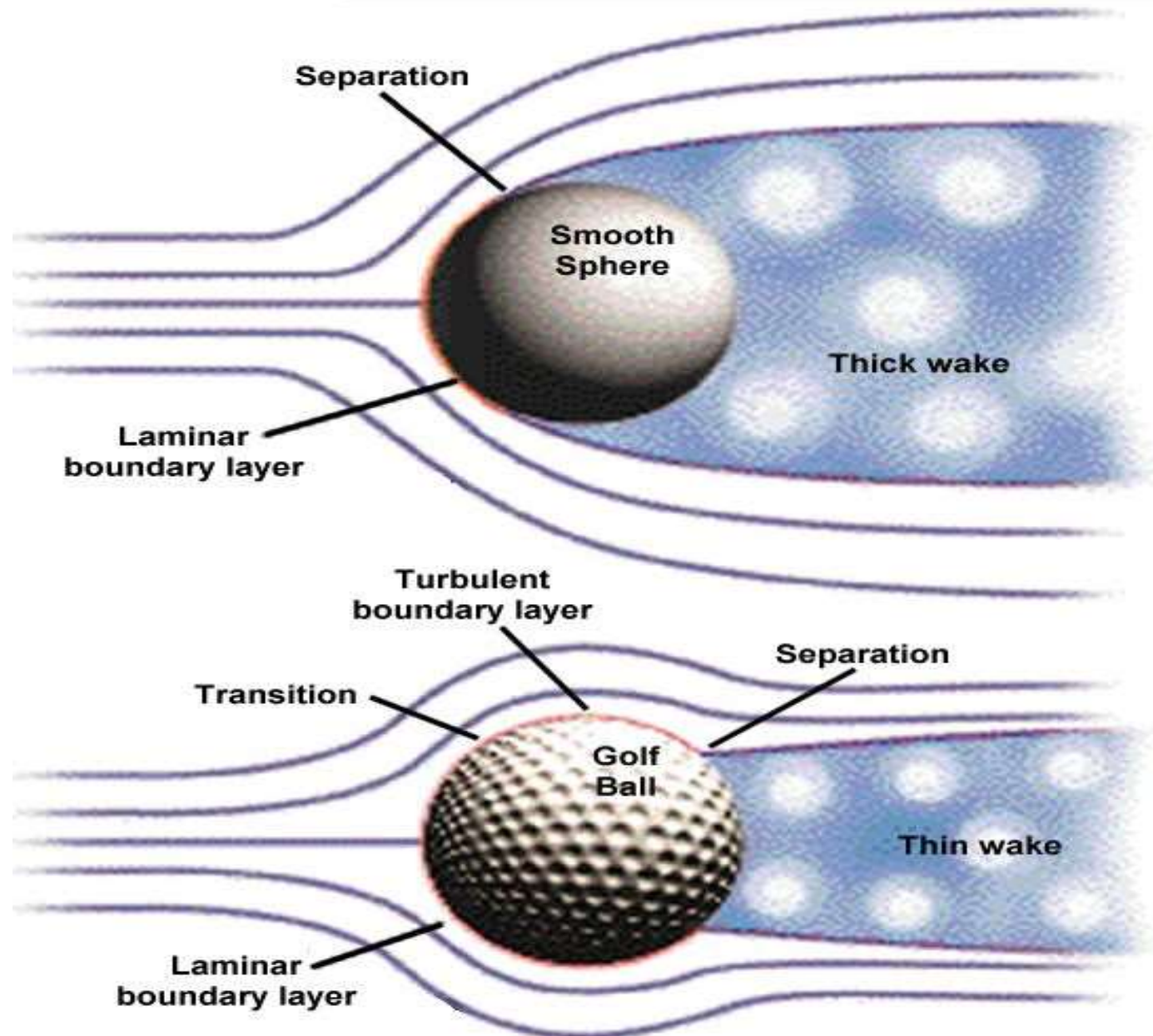
Effects of Heat on Viscous Flow

- The moving fluid has a certain amount of kinetic energy as it flows on the surface. However, the flow velocity is decreased due to friction.

- This decrease in the viscous flow is the loss of kinetic energy of the flow which transforms into internal energy of the fluid.

- This causes the temperature to rise in the fluid and this is called **viscous dissipation**.

Effects of Heat on Turbulent Flow

- Turbulent flows can be more effected by aerodynamic heating.

- If the wall of the surface is being heated that can cause the laminar flow to transform into turbulent flow.

- Consequently, if the wall surface of the airfoil is cooler that may cause the turbulent flow to subside and become laminar flow.

- Thus for blunt bodies, turbulent flow is desirable.

When is Turbulent Flow Desirable?

Very high-
speed flow

Stronger shock wave

Larger heating of the air

Smaller heating of the body
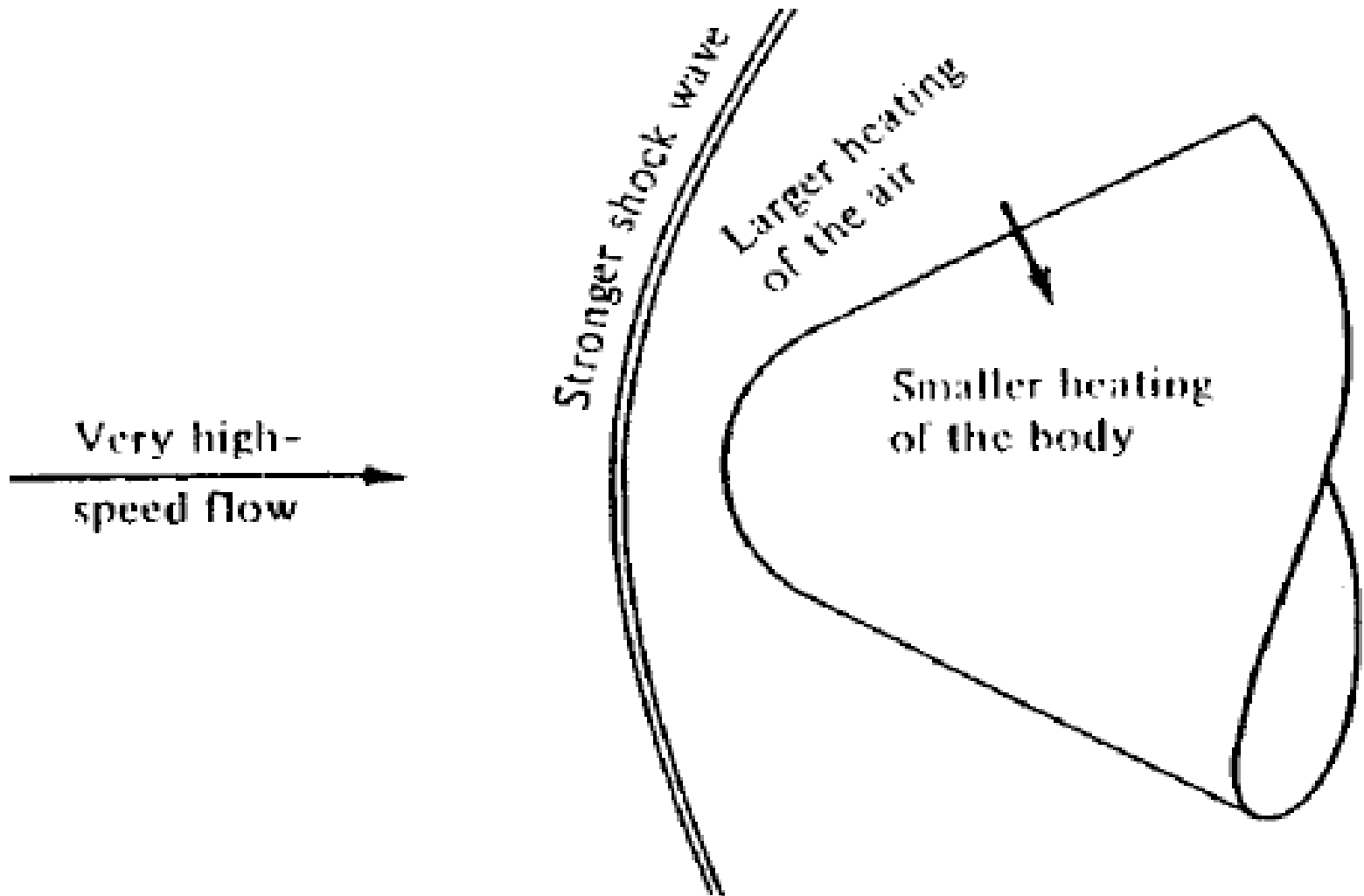
Viscous Flow and Turbulent Flow

- All viscous flows are rotational flows. Thus no velocity potential function exists for viscous flow.

- In viscous flows, the Prandtl number is important. Prandtl number is the property of the gas and it changes as a function of temperature.

$$Pr = \frac{\mu_x c_p}{k} \propto \frac{\text{frictional dissipation}}{\text{thermal conduction}}$$

$Pr_x = \text{constant} = 0.71.$ for air over a reasonable temperature range

Boundary Conditions for Viscous Flow

- 1) No slip condition exists at the airfoil surface. Hence fluid velocity is zero at the surface where **y=0 >>>> u=v=w=0**

- 2) The fluid will have the same temperature as the wall at the surface **y=0 >>> T=T$_w$**

- 3) If T$_w$ is not constant, this means that either the fluid is heated by the wall or the wall is being heated by the fluid. Then the boundary condition is obtained by **Fourier law at Y=0**

How to Analyze a Viscous Flow?

- 1) Take the boundary conditions given at the previous slide into consideration

- 2) Analyze the direction of the flow. Are there any elements that can be omitted?

- 3) Use the Equations given in the next section for calculating velocity and temperature gradients

- 4) Use Shear Stress and Fourier Formula for calculating shear stress and heat flux caused by the fluid on the surface

- 5) Try to predict whether flow separation will occur and at what conditions and intensity will it occur?

Fluid Element in a Viscous Flow
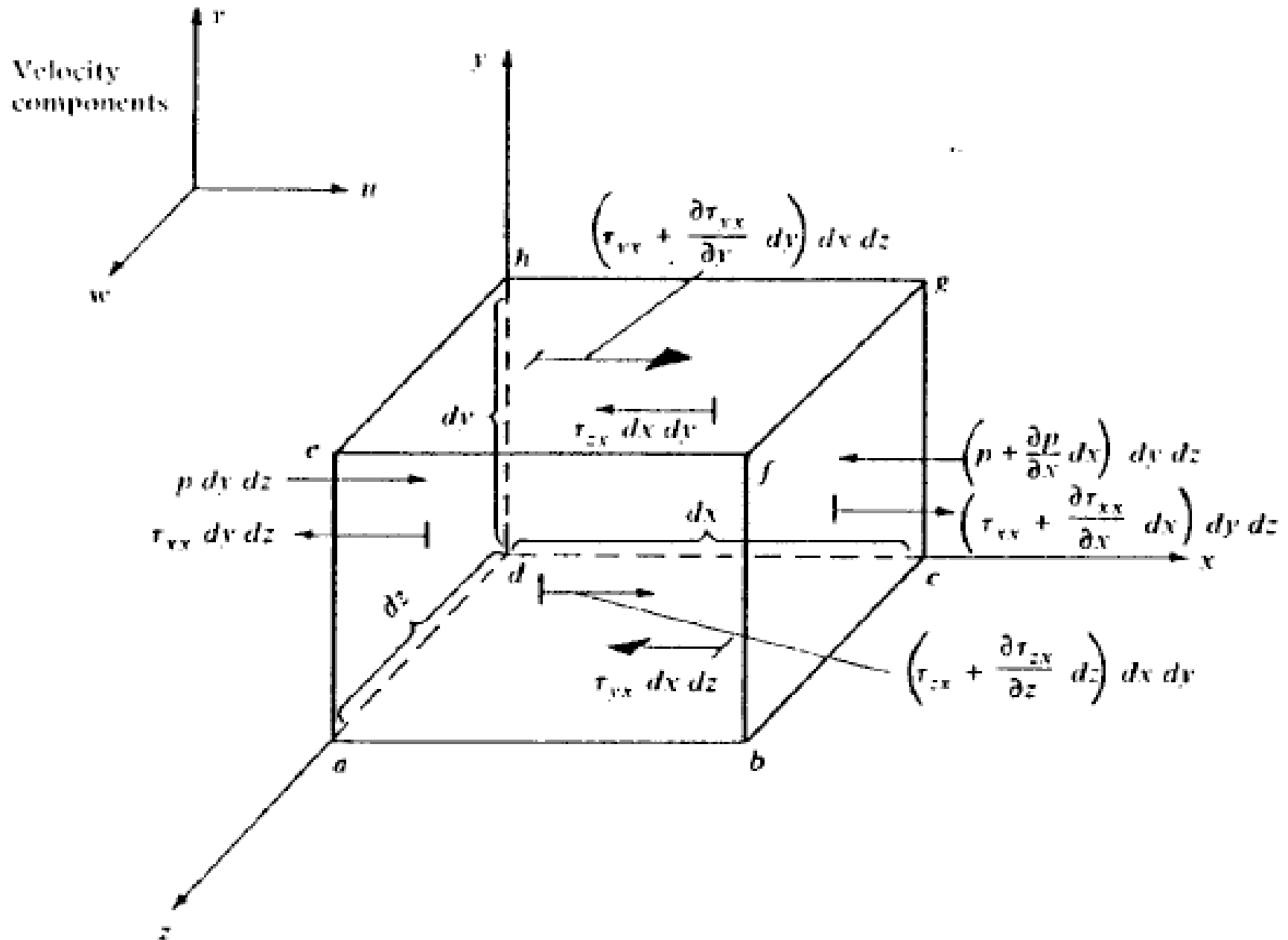
Forces Acting on the Fluid Element

- All of the forces acting on the Fluid element can be summarized by Newton's Second Equation:

$$F_x = ma_x$$

Now lets rewrite the left hand side of the equation for all the forces acting on the x direction:

$$F_x = \left[p - \left(p + \frac{\partial p}{\partial x} dx \right) \right] dy\, dz + \left[\left(\tau_{xx} + \frac{\partial \tau_{xx}}{\partial x} dx \right) - \tau_{xx} \right] dy\, dz$$

$$+ \left[\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy \right) - \tau_{yx} \right] dx\, dz + \left[\left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz \right) - \tau_{zx} \right] dx\, dy$$

$$F_x = \left(-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) dx\, dy\, dz$$

The right hand side of the equation is:

$$m = \rho\, dx\, dy\, dz$$

$$a_x = \frac{Du}{Dt}$$

Navier Stokes Equations

- Navier- Stokes equations are the most famous equations that depict aerodynamic viscous flow.

- In essence, Navier Stokes Equations are the momentum-continuity-energy equations derived in the slide from Newton's Second Law.

- Navier Stokes equations can be applied to any kind of Fluid flow. They depict all of the momentum forces acting on the various variables of the fluid.

Navier Stokes Momentum Equations

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

which is the x component of the momentum equation for a viscous flow. fashion, the y and z components can be obtained as

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

Navier Stokes Momentum Equations

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial u}{\partial x}\right)$$

$$+ \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]$$

$$+ \frac{\partial}{\partial y}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right]$$

$$\rho \frac{\partial w}{\partial t} + \rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$+ \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right] + \frac{\partial}{\partial z}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial w}{\partial z}\right)$$

Energy Equations

Rate of change
of energy inside $=$ net flux of heat into element $+$ rate of work done on element due to pressure and stress forces on surface

$$A \quad = \quad B \quad + \quad C$$

$$\rho \frac{D(e + V^2/2)}{Dt} = \rho \dot{q} + \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right)$$

$$+ \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \nabla \cdot p\mathbf{V} + \frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y}$$

$$+ \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{xy})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(v\tau_{zy})}{\partial z}$$

$$+ \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial(w\tau_{yz})}{\partial y} + \frac{\partial(w\tau_{zz})}{\partial z}$$

Navier–Stokes Equations
3 – dimensional – unsteady

Coordinates: (x, y, z)

Velocity Components: (u, v, w)

Time : t Pressure: p

Density: ρ Stress: τ

Total Energy: Et

Heat Flux: q

Reynolds Number: Re

Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$
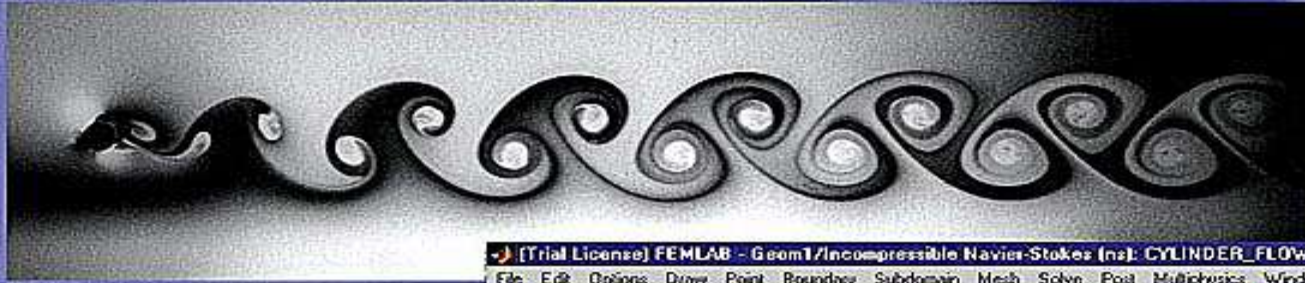
X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho u v)}{\partial y} + \frac{\partial(\rho u w)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho u v)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho v w)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho u w)}{\partial x} + \frac{\partial(\rho v w)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$
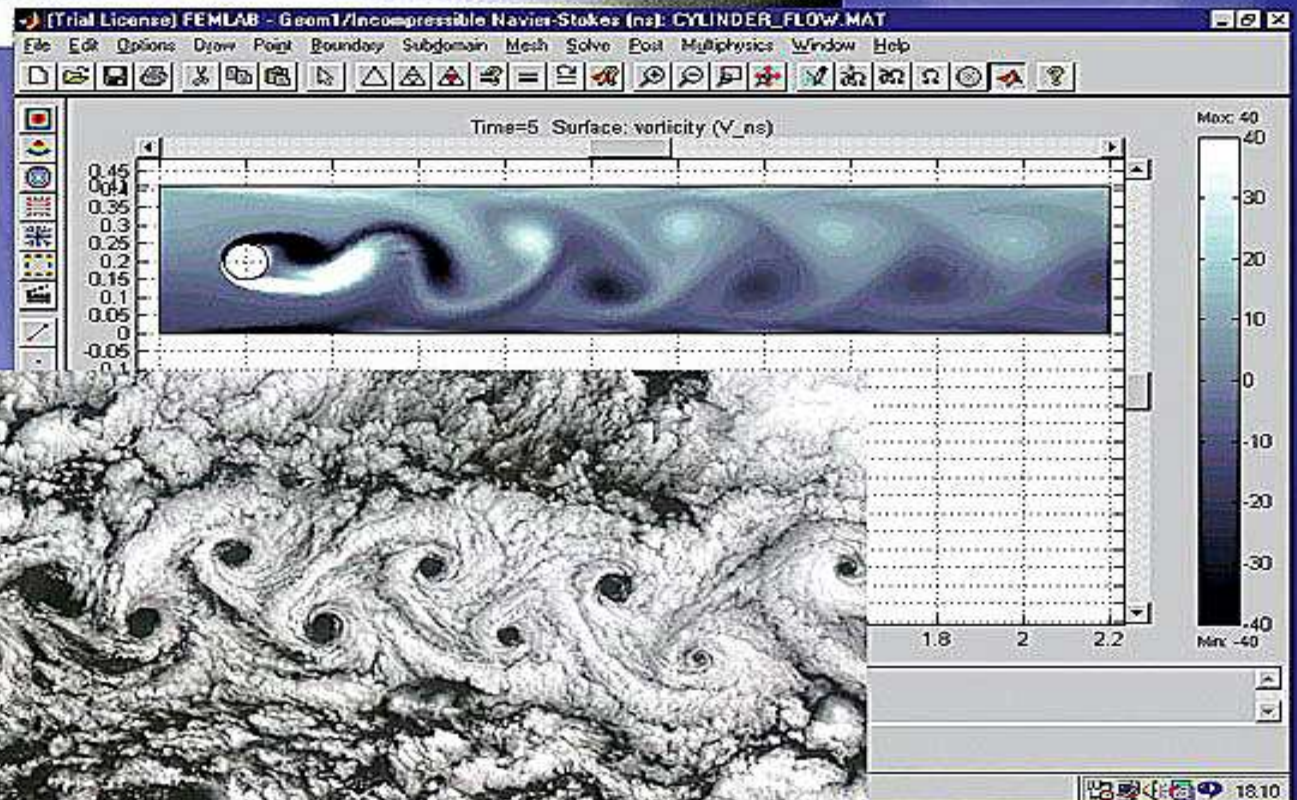
Energy:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(u E_T)}{\partial x} + \frac{\partial(v E_T)}{\partial y} + \frac{\partial(w E_T)}{\partial z} = -\frac{\partial(u p)}{\partial x} - \frac{\partial(v p)}{\partial y} - \frac{\partial(w p)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$

$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u \tau_{xx} + v \tau_{xy} + w \tau_{xz}) + \frac{\partial}{\partial y}(u \tau_{xy} + v \tau_{yy} + w \tau_{yz}) + \frac{\partial}{\partial z}(u \tau_{xz} + v \tau_{yz} + w \tau_{zz})\right]$$

Graphic Representation of a Navier Stokes Solution

Solutions to the Viscous Flow

- In order to solve a viscous flow variables, you first need to write the Continuity Equation, Momentum Equations as well as the Energy Equation relevant to the flow at hand.

- You simplify some terms thinking about the terms that may be cancelled such as delta t=0 for steady flow or 1 d flow with v=w=0 etc.

- Once you have solved for the variables such as speed, density, temperature and pressure, you can calculate the shear stress easily

Modeling of the Navier Stokes Equations

U Magnitude

0.00   21.6   43.1   64.7   86.2
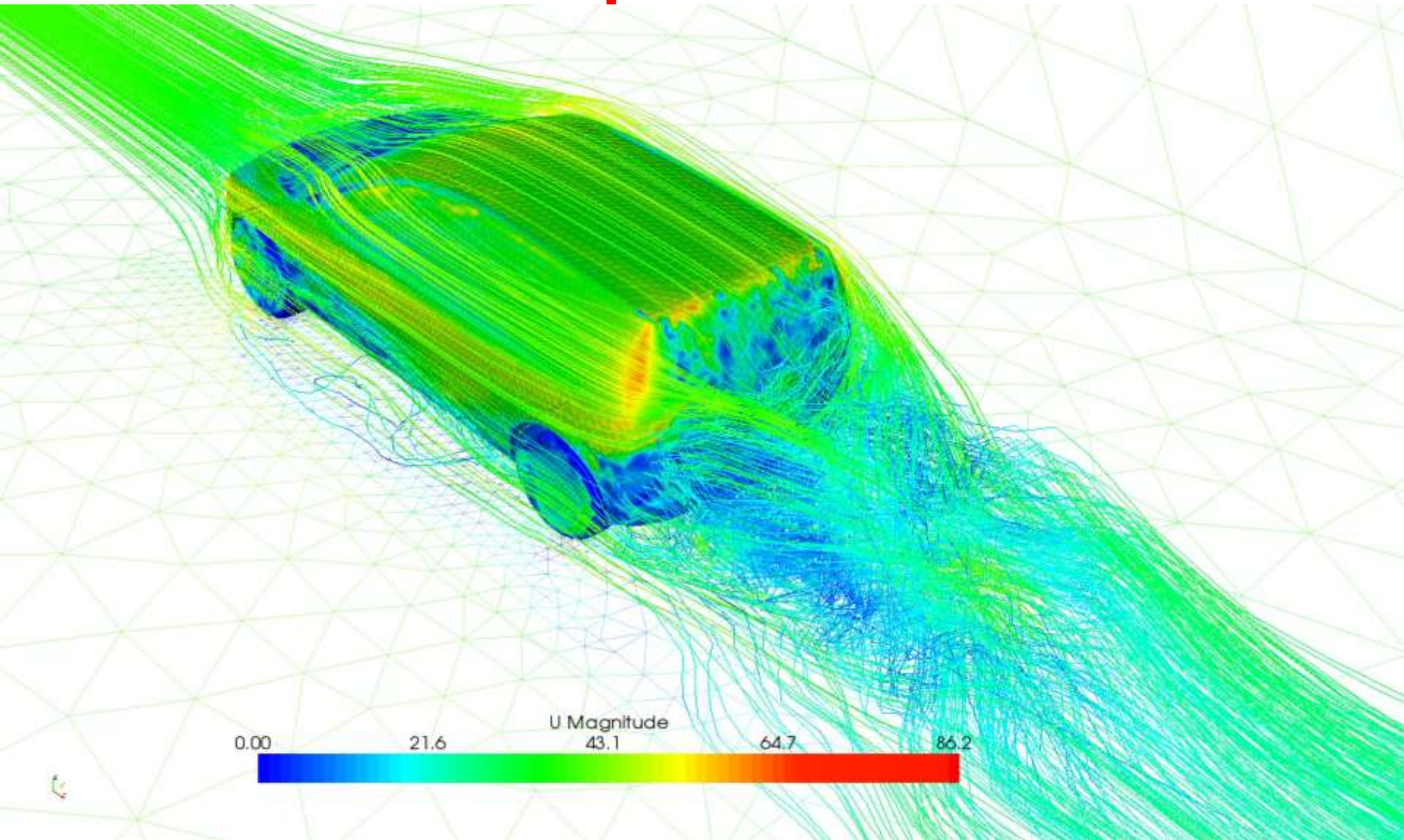
Shear Stress in a Fluid Element
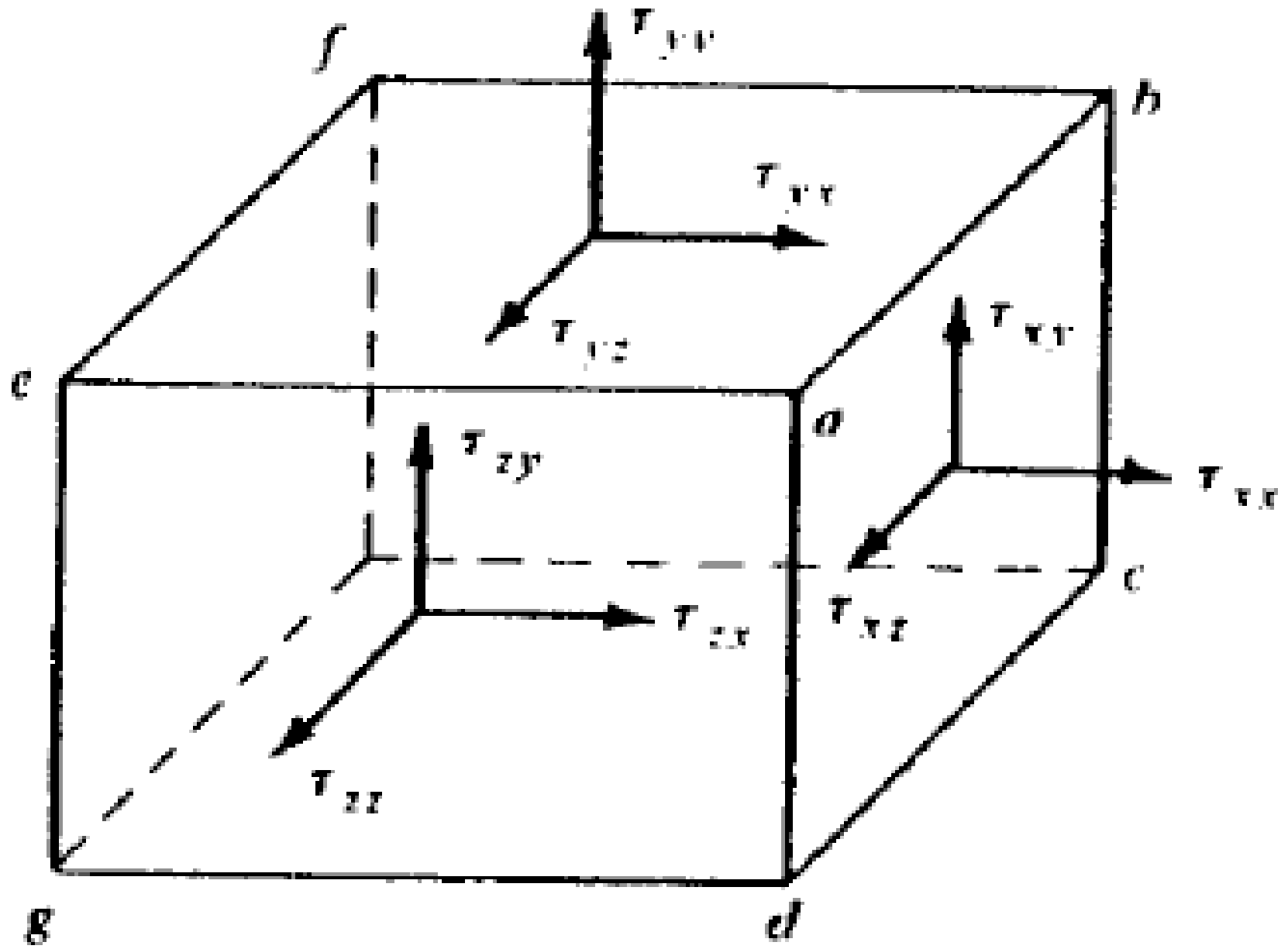
Shear Stress

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

Shear Stress

$$\lambda = -\frac{2}{3}\mu \qquad \lambda \text{ is called the } \textit{bulk viscosity coefficient}$$

$$\tau_{xx} = \lambda (\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial u}{\partial x}$$

$$\tau_{yy} = \lambda (\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial v}{\partial y}$$

$$\tau_{zz} = \lambda (\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial w}{\partial z}$$

is the pressure force. However, if the gradients in velocity $\partial u/\partial x$, $\partial v/\partial y$, and $\partial w/\partial z$ are *extremely large* on the faces of the fluid element, there can be a meaningful viscous-induced normal force on each face which acts *in addition to* the pressure. These normal stresses act to compress or expand the fluid element, hence changing its volume. Recall

Basic Solutions to Viscous Flow

- There are two basic solutions to Viscous Flow Equations.

- These two solutions are solved for analytically by using the Navier Stokes equations.

- These unsolvable equations are solved by making some terms come to zero due to cancellation of forces.

- The two main basic solutions to Viscous flow are:

a) Couette Flow

b) Poisoille Flow

Couette Flow

- The Couette flow can be best described as the *flow between two plates, where the top plate is moving and the bottom plate is stationary*.
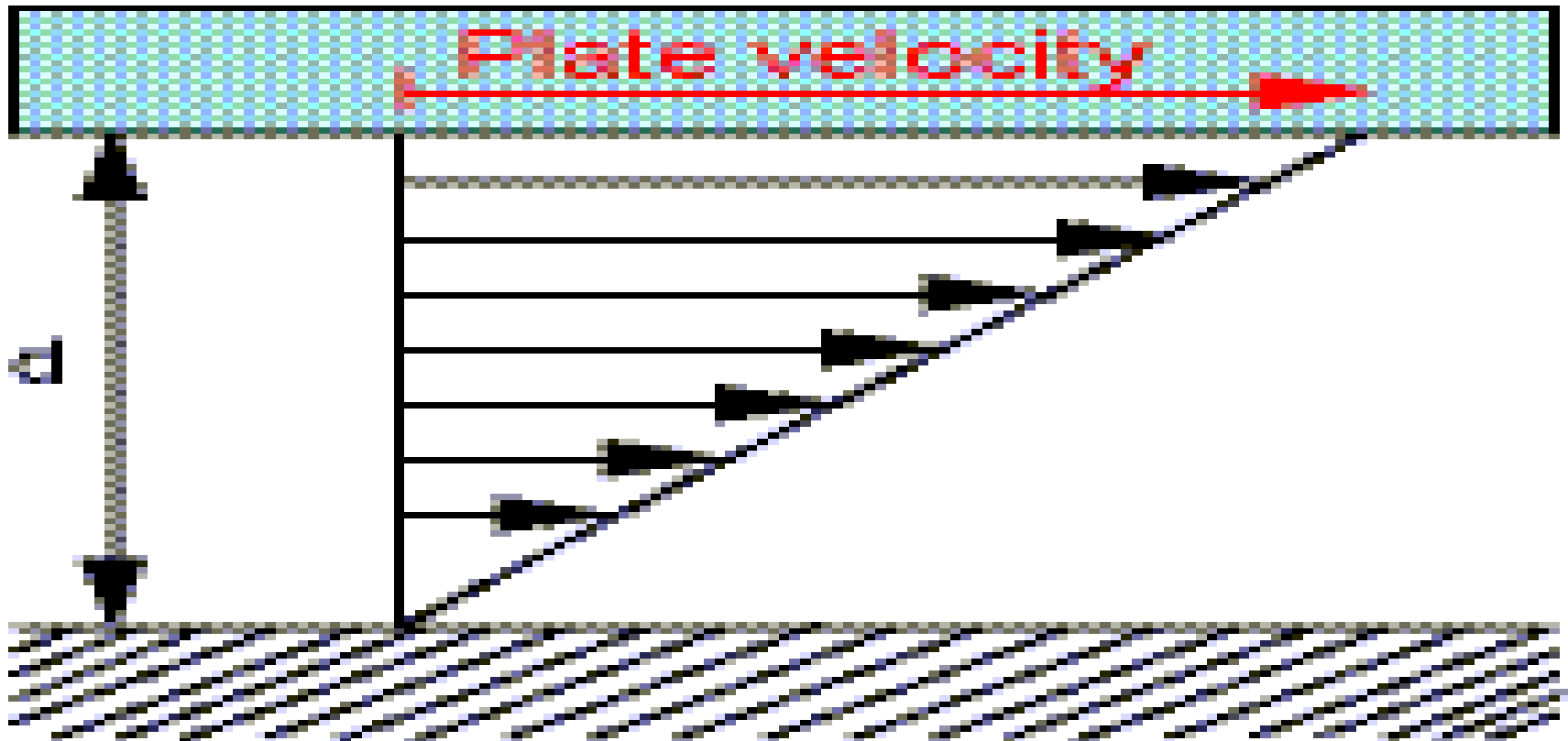
Couette Flow

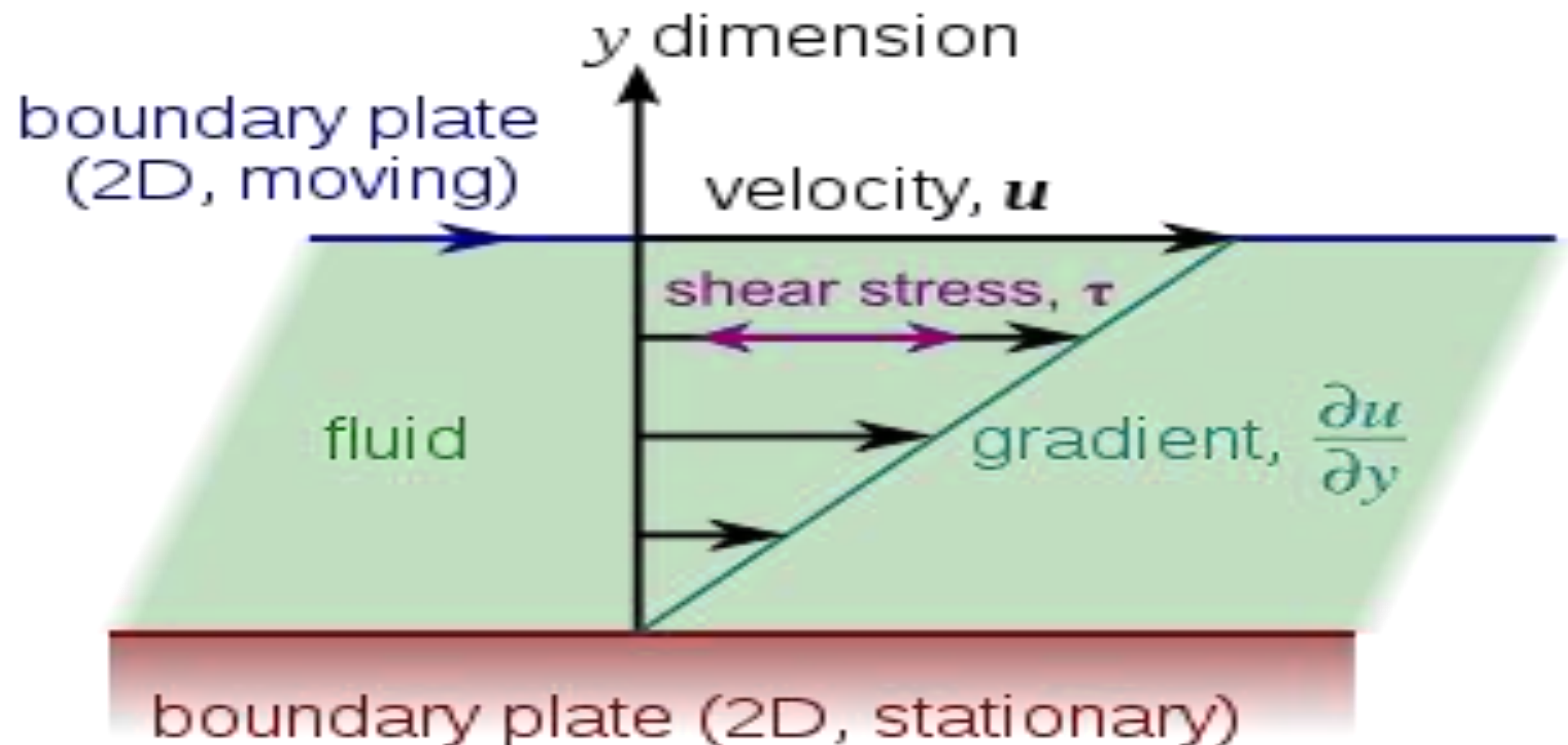- The driving force for this flow is the motion of the upper plate dragging the flow along with it through the mechanism of friction.

- The upper plate is is exerting a shear stress acting toward the fluid at y=D that is causing the fluid to move toward the right.

Couette Flow

- The only changes in the flow take place in the y direction. Nothing changes in the x and z direction.

- The flow velocity is zero at the bottom where the stationary plate is. The flow velocity gradually increases as d is increased and reaches maximum at the top where the moving plate is.

Moving Wall

Plate velocity

d
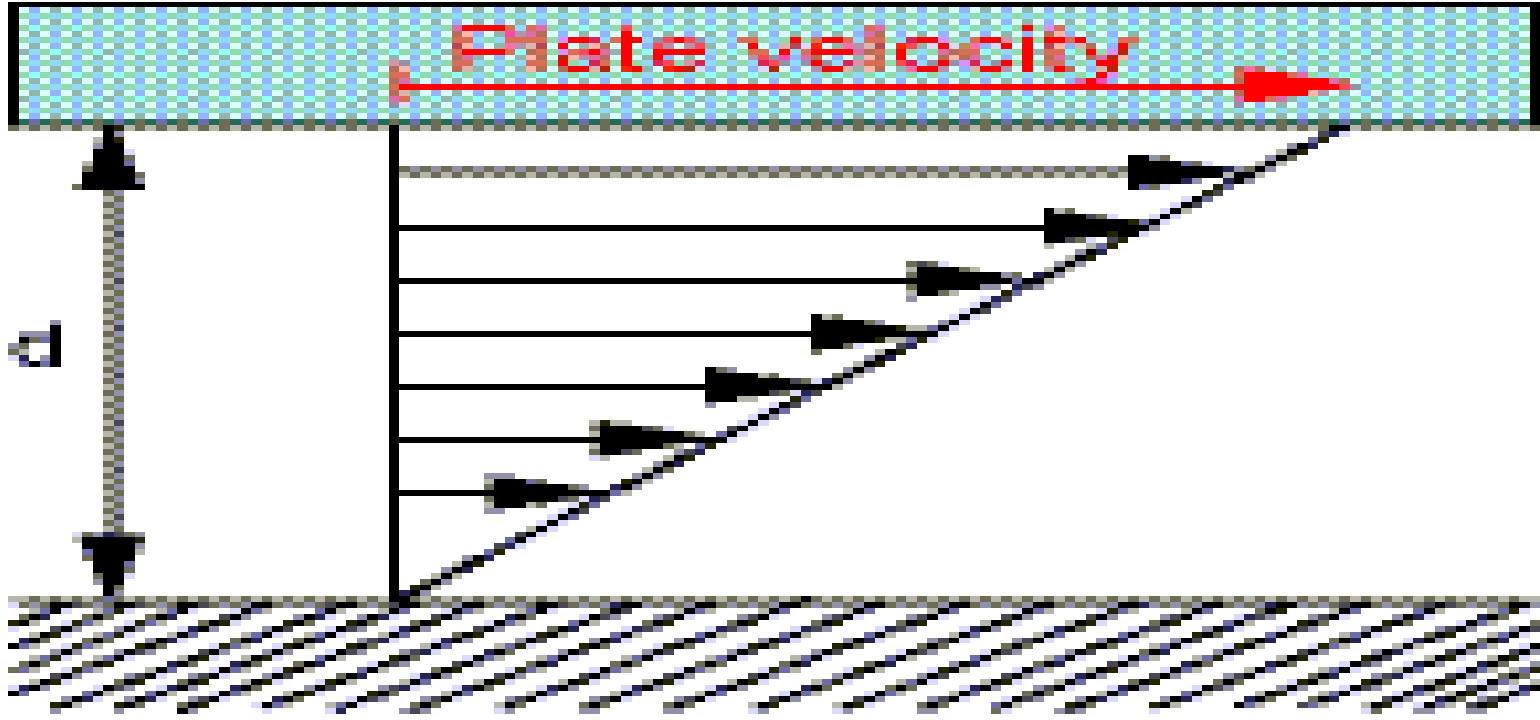
Couette Flow

- The kinetic energy of the flow is dissipated partially through friction and turned to internal energy in the fluid. This is called **viscous dissipation.**

- The heat flux is expressed by the Fourier Law.

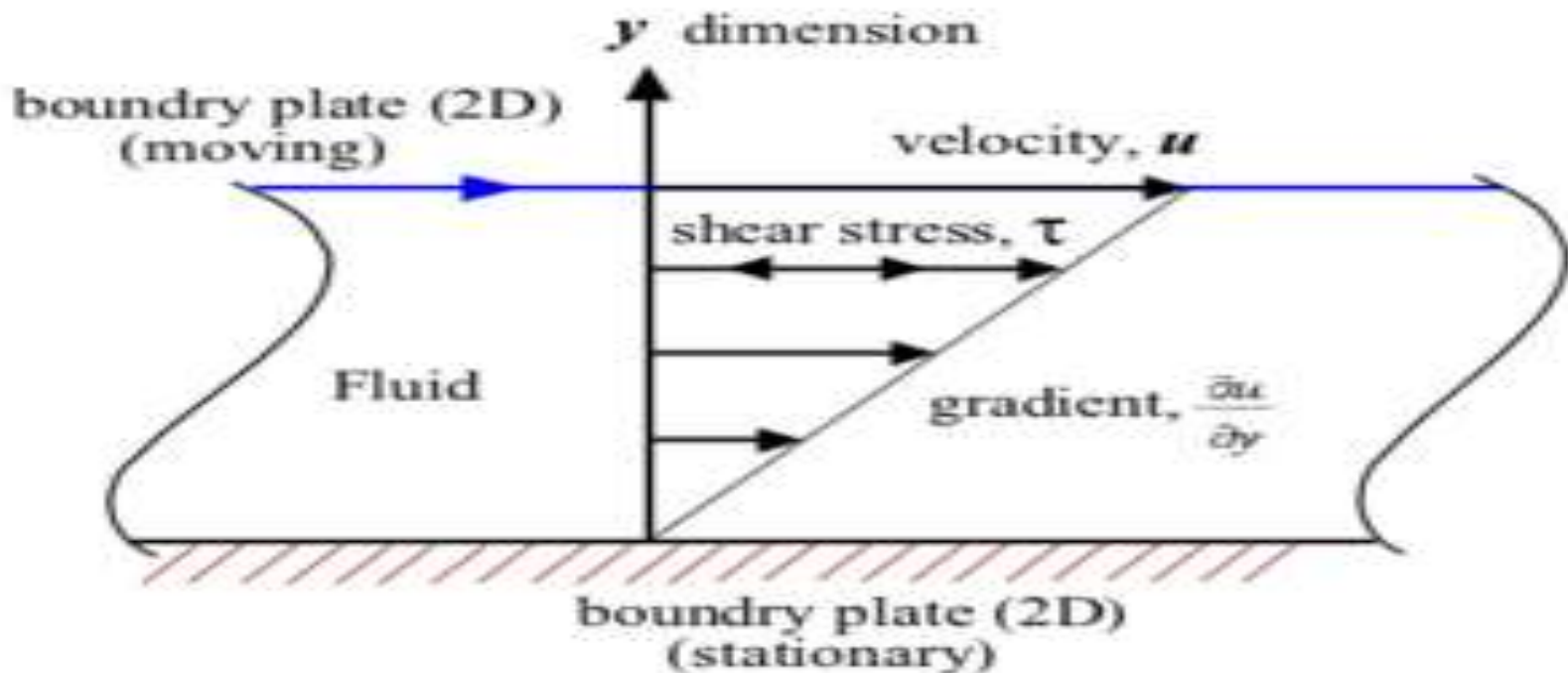$$q = -k \frac{\partial T}{\partial y}$$

- If the stationary plate and the moving plate have different temperatures, there will be temperature gradients in the flow.

Couette Flow

- In this flow, all flow properties **only change with y**.

- Thus, all partial derivatives in the Navier Stokes equations **with respect to x and z are zero.**

- The flow is steady, so all **time derivatives are zero.**

Navier–Stokes Equations
3 – dimensional – unsteady

Coordinates: (x, y, z)

Velocity Components: (u, v, w)

Time : t Pressure: p

Density: ρ Stress: τ

Total Energy: Et

Heat Flux: q

Reynolds Number: Re

Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$

$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})\right]$$

Navier Stokes Momentum Equations

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial u}{\partial x}\right)$$

$$+ \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]$$

$$+ \frac{\partial}{\partial y}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right]$$

$$\rho \frac{\partial w}{\partial t} + \rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$+ \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right] + \frac{\partial}{\partial z}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial w}{\partial z}\right)$$

Couette Flow Equations

- The X Momentum Equation Becomes:

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 0$$

- The Y Momentum Equation Becomes:

$$\frac{\partial p}{\partial y} = 0$$

- The Energy Equation Becomes:

$$\frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu u \frac{\partial u}{\partial y}\right) = 0$$

Couette Flow

- The Couette Flow is a constant pressure flow since pressure doesn't change in the x direction or y direction due to the equations.

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

- Hence, Couette Flow happens due to shear stress exerted on the flow.

- All flows happen either due to pressure difference or due to shear stress. No other form of flow exists in the world.

Conclusions About Couette and Viscous Flow

- Aerodynamic heating increases as the flow velocity increases. This is especially important for hypersonic vehicles as extreme temperatures are reached due to viscous dissipation.

- Shear stress decreases as the distance between the two sides of the flow increases.

Incompressible Constant Couette Flow

- The Couette Flow equations are:

v=w=0, Also density and Temperature are Constant

$$\frac{\partial u}{\partial x} = \frac{\partial T}{\partial x} = \frac{\partial P}{\partial x} = 0$$

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 0$$

$$\frac{\partial p}{\partial y} = 0$$

$$\frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu u \frac{\partial u}{\partial y}\right) = 0$$
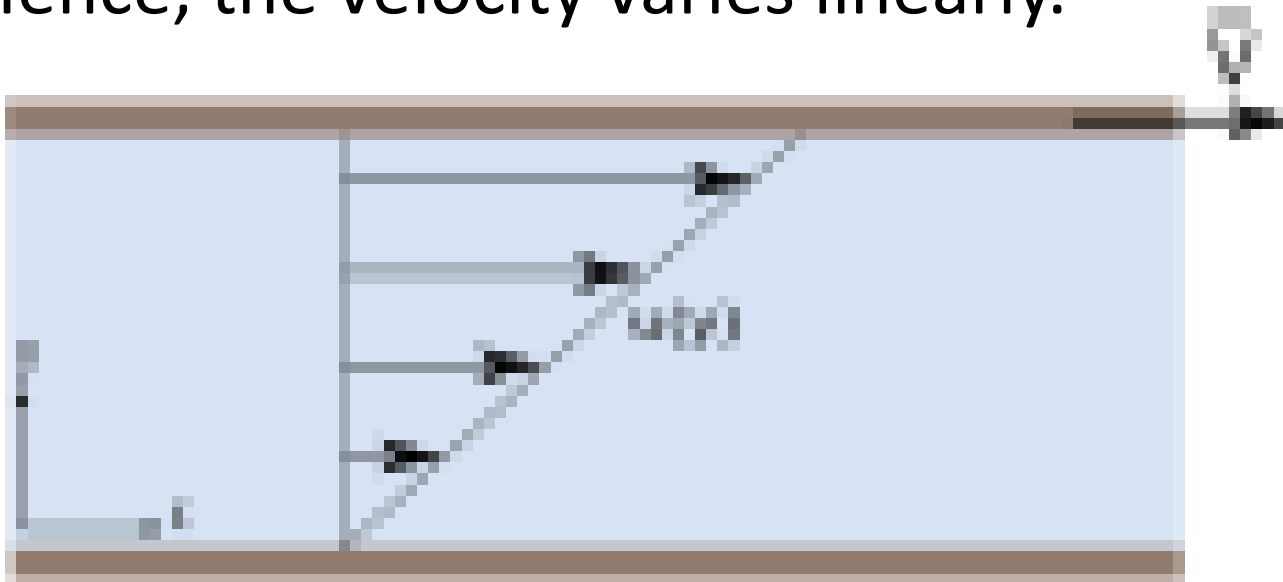
Incompressible Couette Flow

- If we integrate the X Momentum equation:

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 0$$

- We will get:

$$u = u_e\left(\frac{y}{D}\right)$$

- Hence, the velocity varies linearly.

Incompressible Couette Flow

- Once the velocity profile is learned, we can obtain shear stress at any point in the flow.

- The shear stress formula in viscous flows:

$$\tau = \mu \frac{\partial u}{\partial y}$$

- The velocity for incompressible flow is:

$$u = u_e \left(\frac{y}{D}\right)$$

- Hence, the final result for shear stress is:

$$\tau = \mu \left(\frac{u_e}{D}\right)$$

Physical Trends with Viscous Flows

- As the speed of the flow increases, the shear stress increases for viscous flows. ***For Couette Flow this increase is linear.***

- As D increases (the distance between the two opposing sides): the shear stress decreases in Viscous Flows. This is because bigger D means more volume for the shear pressure to dissipate. ***For Couette Flows, shear stress decreases in inverse linear proportion to D.***
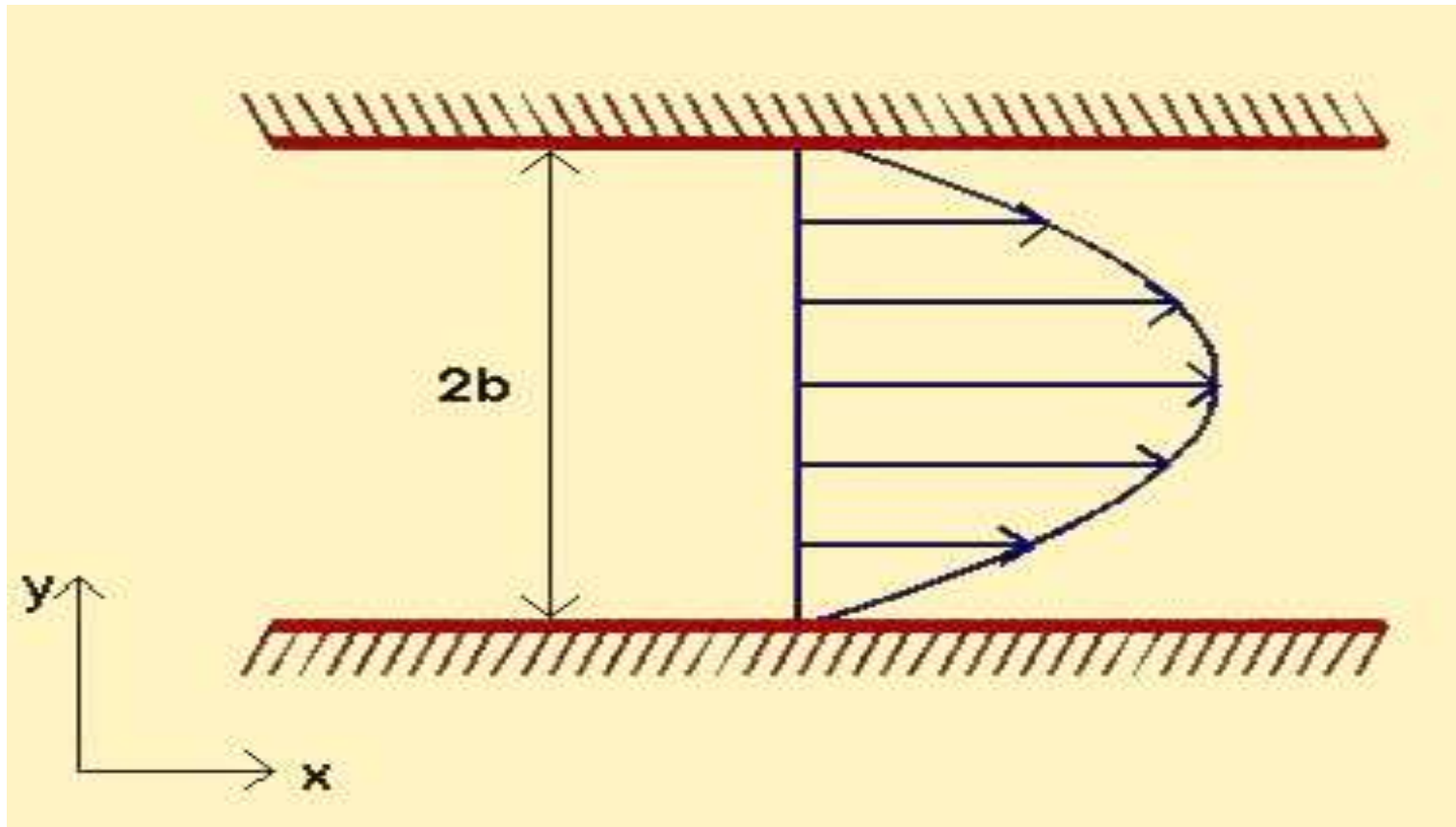
Poiseuille Flow

- Poiseuille Flow is the viscous flow between two stationary plates.
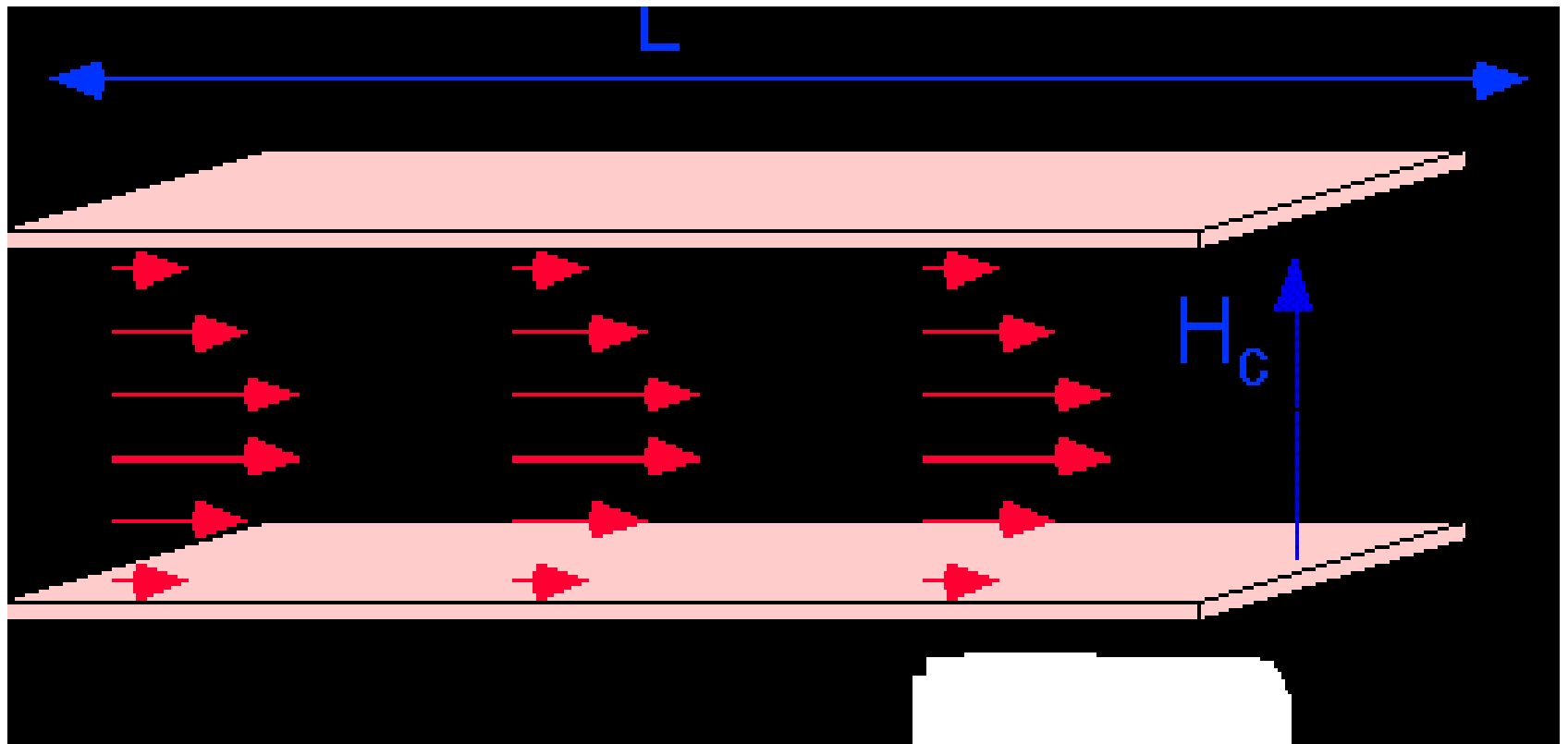
- Unlike Couette Flow, both plates are stationary.
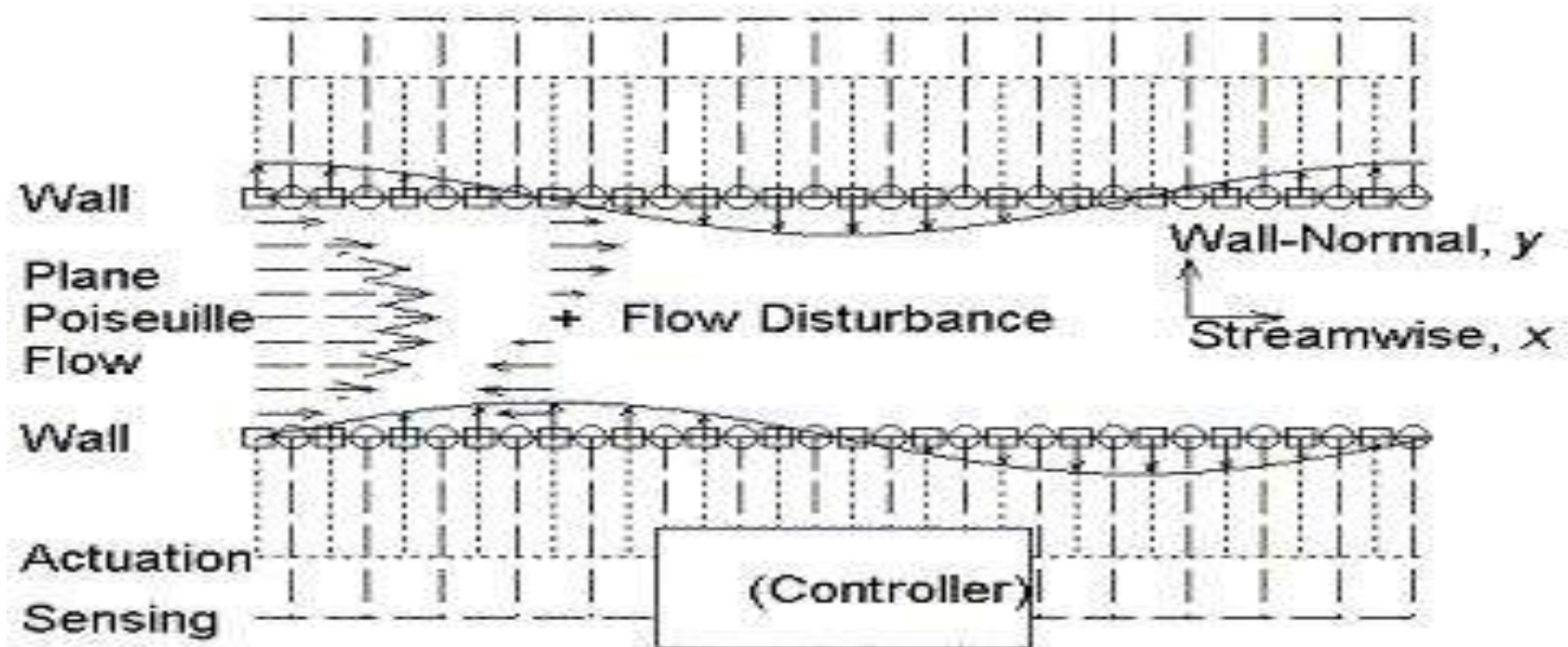
Poiseuille Flow

- In the two dimensional Poiseuille Flow, the velocity of the flow changes in the y direction.
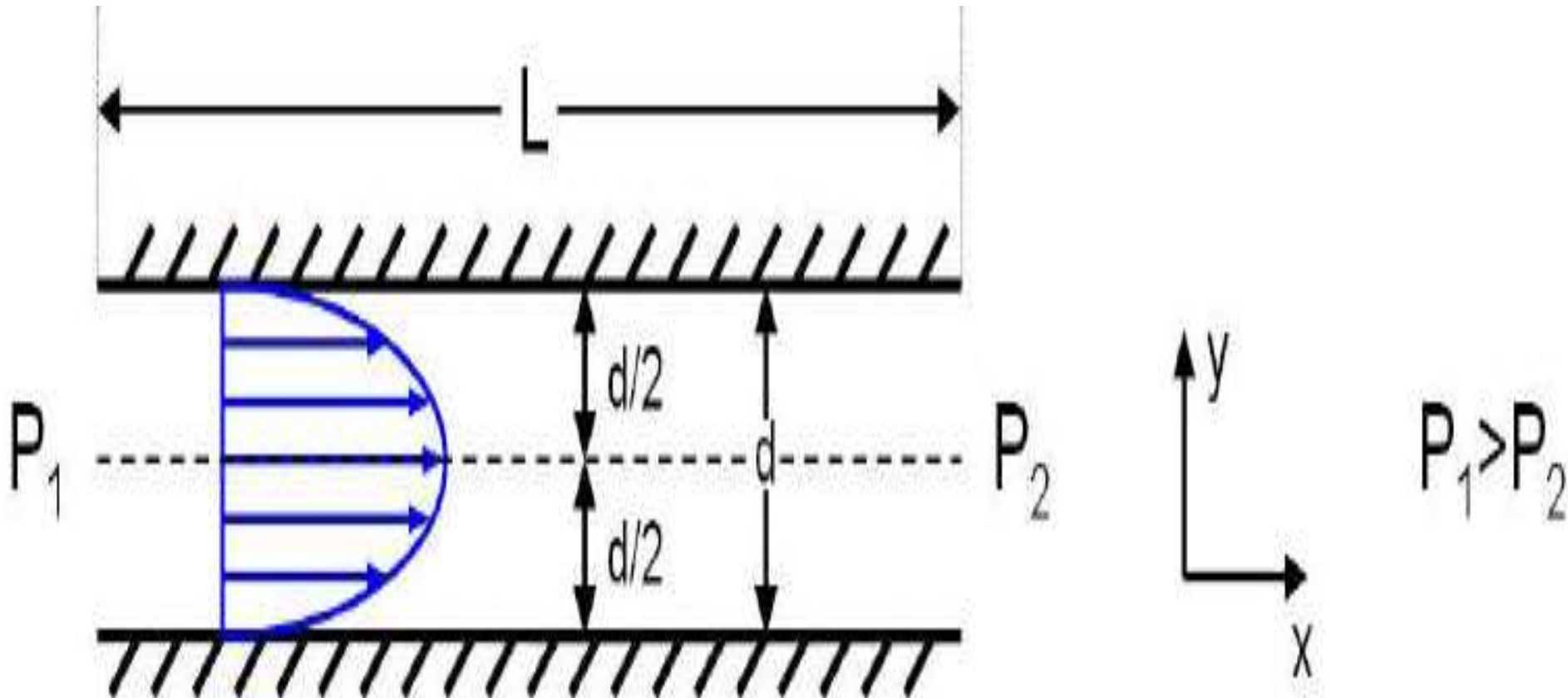- Thus, the flow velocity is independent of x and hence u=u(y) and v=w=0
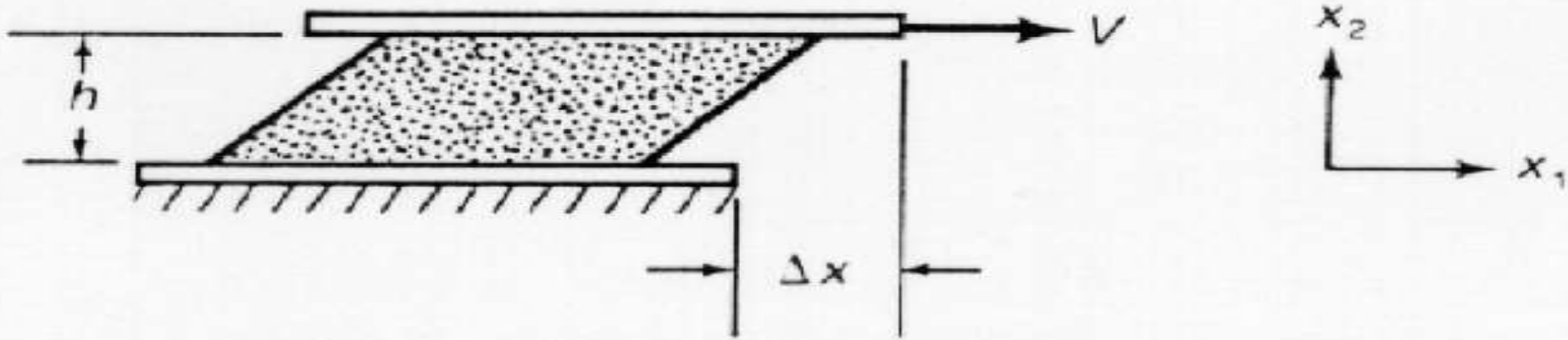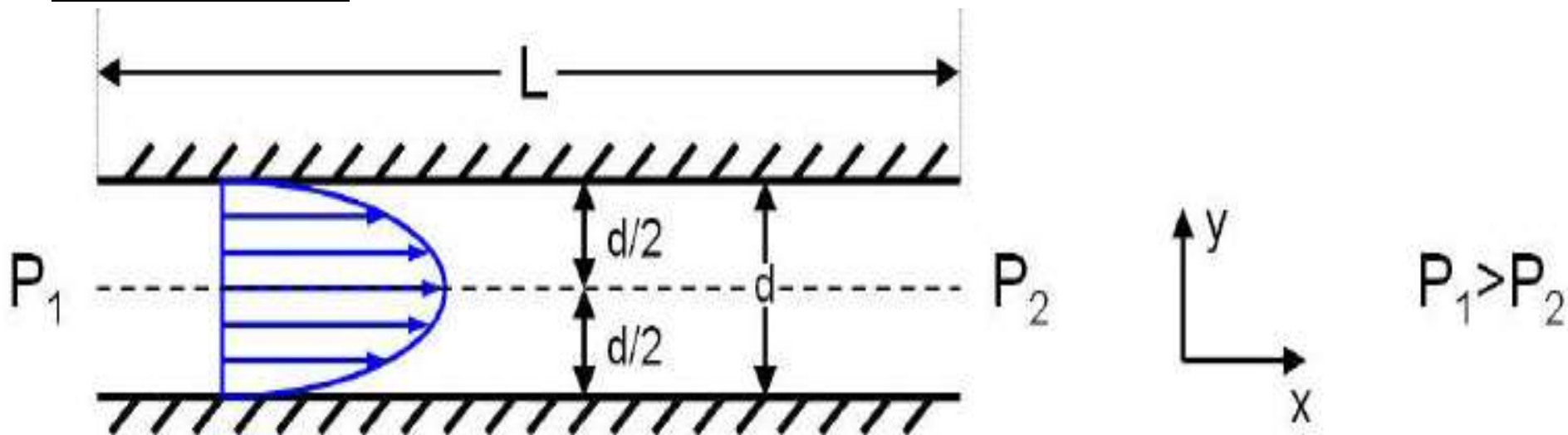
Poiseuille Flow

Poiseuille Flow

- The speed of the flow becomes maximum at the midpoint of the flow as the flow velocity shows a parabolic change with the minimum points being at the stationary plates where the viscous flow velocity is zero
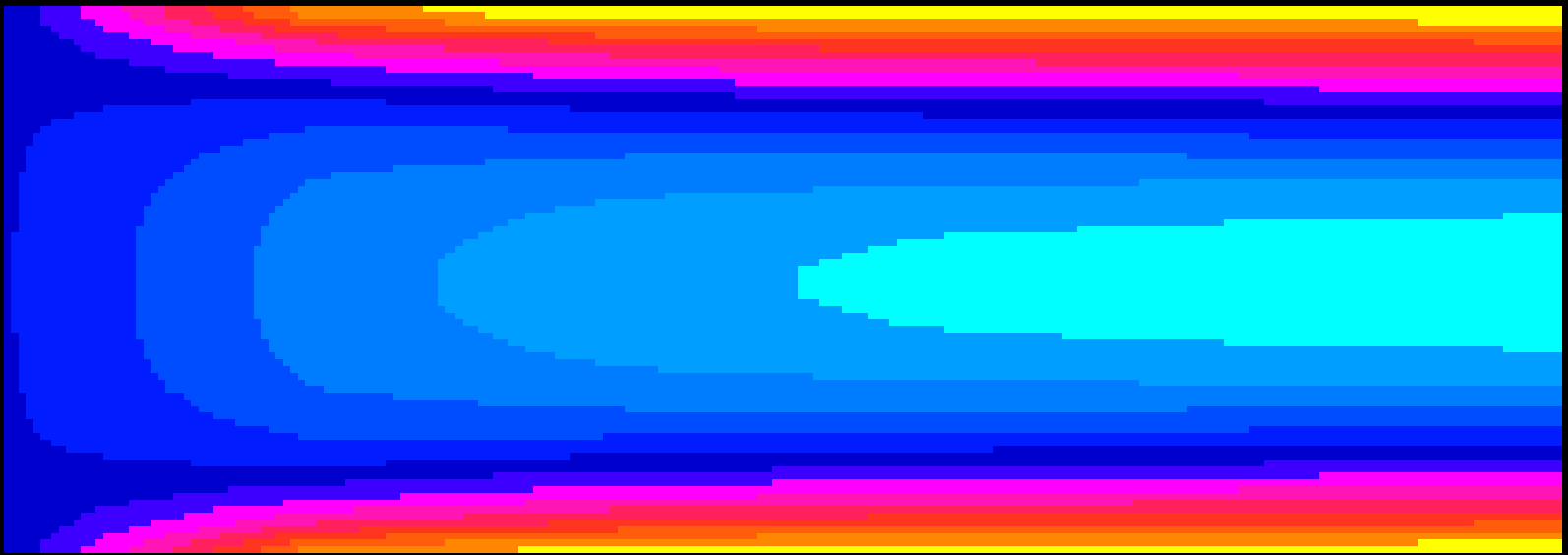
Poiseuille Flow

- Unlike the Couette Flow which is maintained only **by shear stress** of the upper plate dragging the fluid, The Poiseuille flow happens only **due to the pressure difference.**

Poiseuille Flow

- The velocity distribution is parabolic. The velocities at the boundaries of the plates are zero.

- The velocity increases as it reaches the center of the flow and then it starts to decrease again until it becomes zero at the borders

Navier Stokes Momentum Equations

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial u}{\partial x}\right)$$

$$+ \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]$$

$$+ \frac{\partial}{\partial y}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right]$$

$$\rho \frac{\partial w}{\partial t} + \rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$+ \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right] + \frac{\partial}{\partial z}\left(\lambda \nabla \cdot \mathbf{V} + 2\mu \frac{\partial w}{\partial z}\right)$$

Poiseuille Flow Equations

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

$$\frac{\partial p}{\partial y} = 0$$