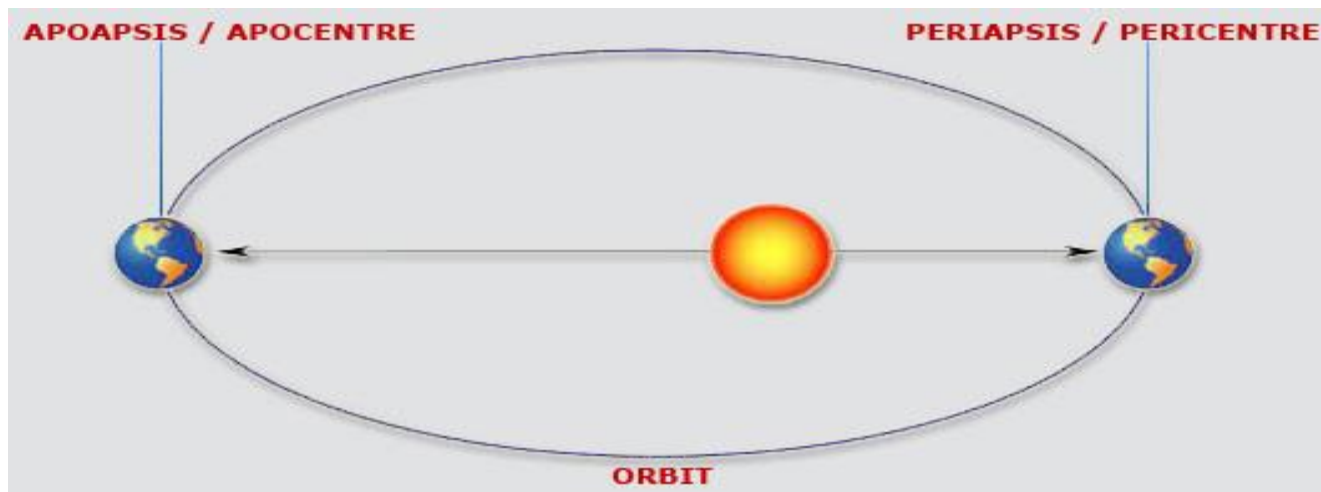


# Satellite Orbit Dynamics

Dr Ugur GUVEN

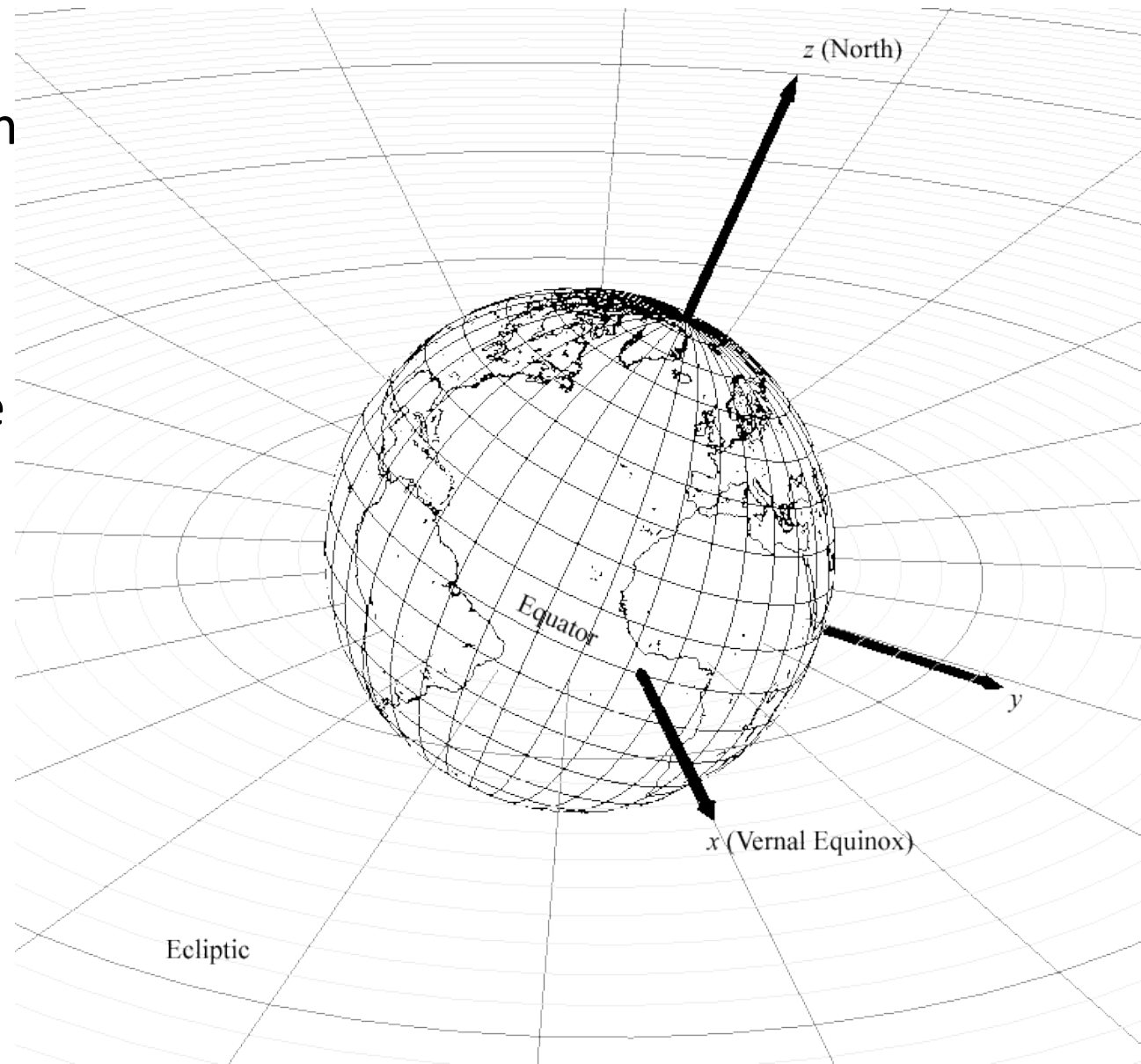
# Apoapsis - Periapsis

- Apoapsis is the furthest position in an orbit around a primary (apogee, aphelion etc)
- Periapsis is the nearest position in an orbit around a primary (perigee, perihelion etc)

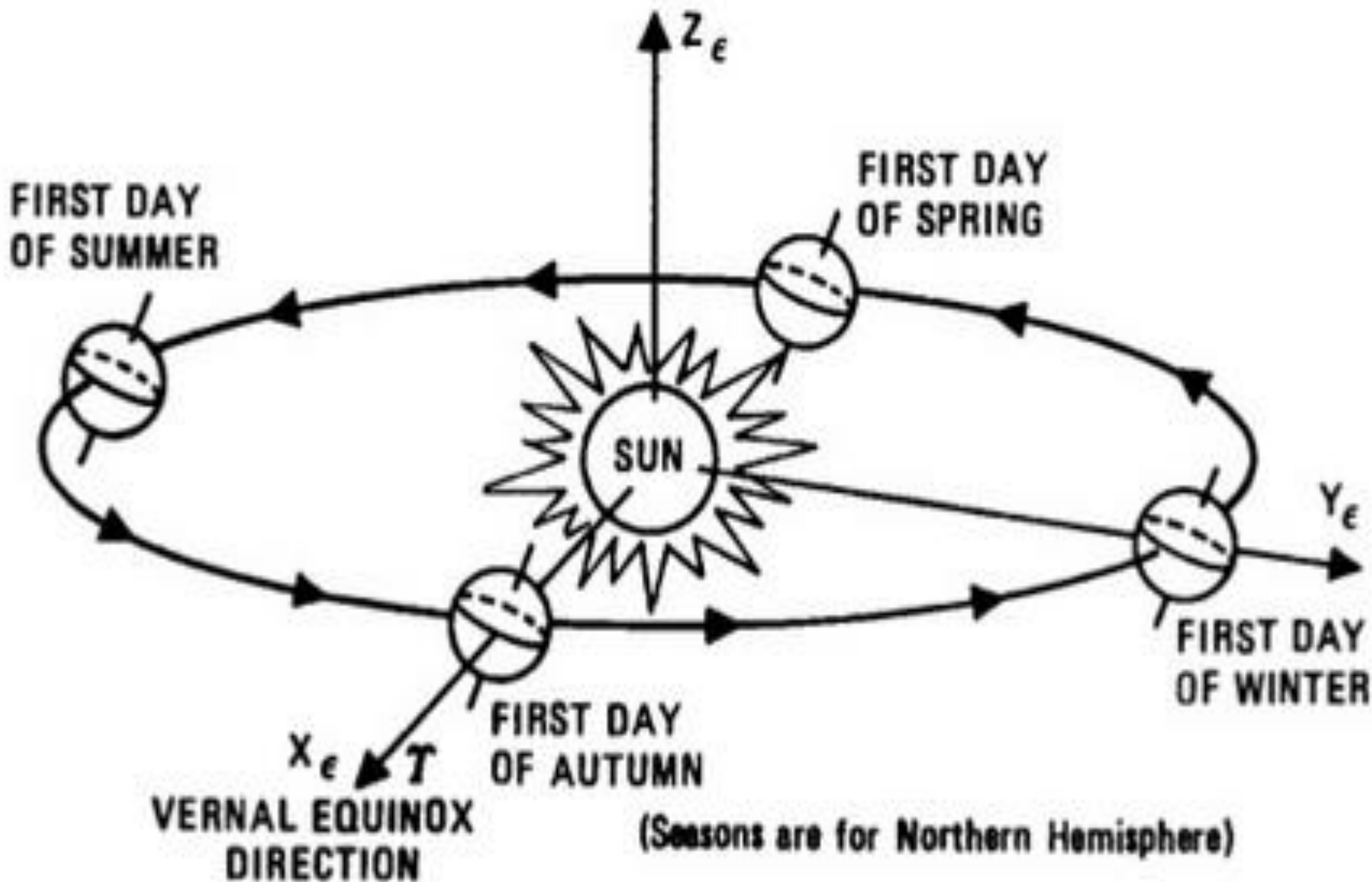


# Vernal Equinox – Equatorial Plane

- The vernal equinox is an imaginary point in space which lies along the line representing the intersection of the Earth's equatorial plane and the plane of the Earth's orbit around the Sun or the *ecliptic*. Sun passes through the vernal equinox, about March 21, marking the beginning of spring in the Northern Hemisphere



# Heliocentric Coordinate System

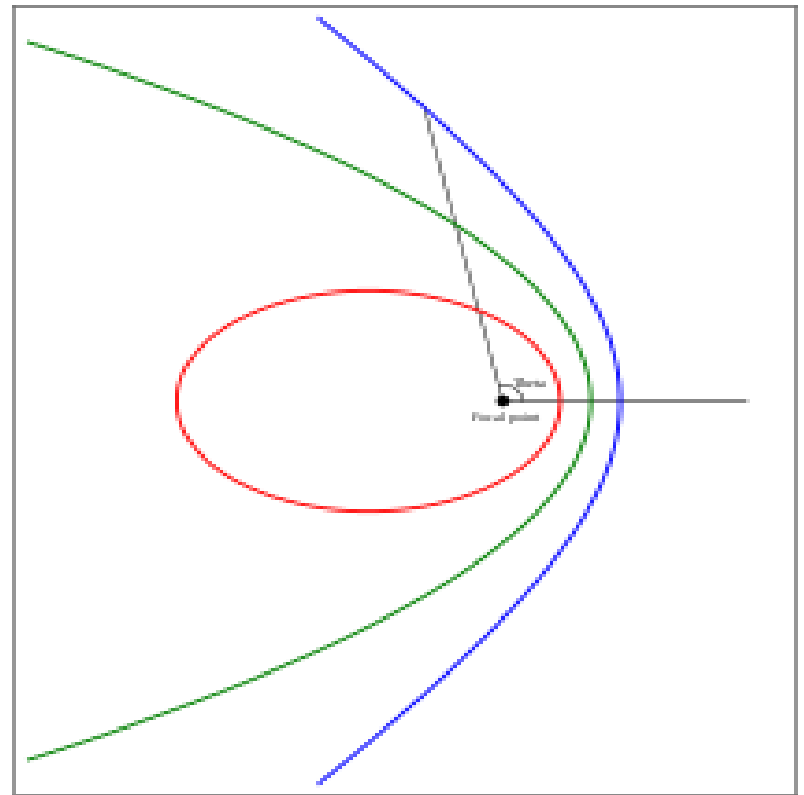


# Epoch

- **Epoch** is a moment in time used as a reference point for some time-varying astronomical quantity, such as the celestial coordinates or elliptical orbital elements of a celestial body, because these are subject to perturbations and vary with time.
- 86 50.28438588 as Epoch in Julian time is:
- The epoch year (1986) and 50.28438588 as the Julian day fraction meaning a little over 50 days after January 1, 1986. The resulting time of the vector would be **1986/050:06:49:30.94**.
- Start with **50.28438588 days** (Days = 50)  
50.28438588 days - 50 = 0.28438588 days  
0.28438588 days x 24 hours/day = 6.8253 hours (**Hours = 6**)  
6.8253 hours - 6 = 0.8253 hours  
0.8253 hours x 60 minutes/hour = 49.5157 minutes (**Minutes = 49**)  
49.5157 - 49 = 0.5157 minutes  
0.5157 minutes x 60 seconds/minute = **30.94 seconds** (Seconds = 30.94)

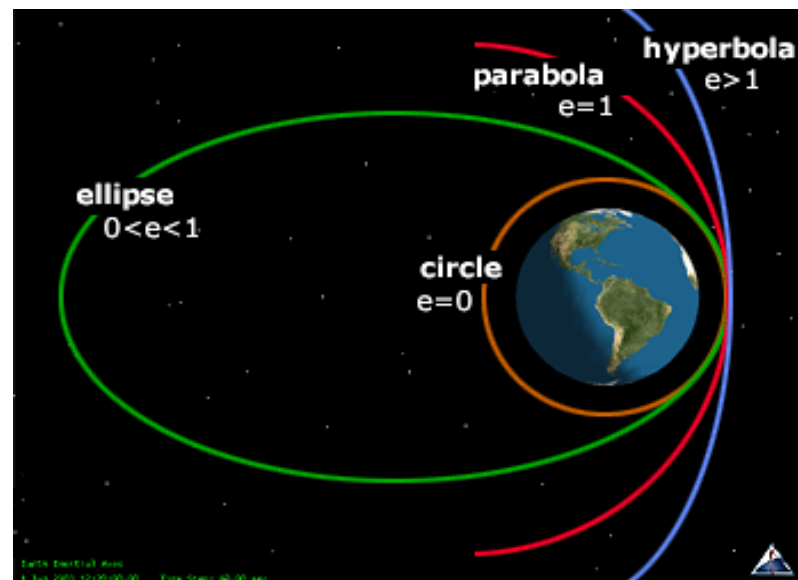
# Orbital Eccentricity

- The **orbital eccentricity** of an astronomical object is a parameter that determines the amount by which its orbit around another body deviates from a perfect circle. **A value of 0 is a circular orbit, values between 0 and 1 form an elliptical orbit, 1 is a parabolic escape orbit, and greater than 1 is a hyperbola.**



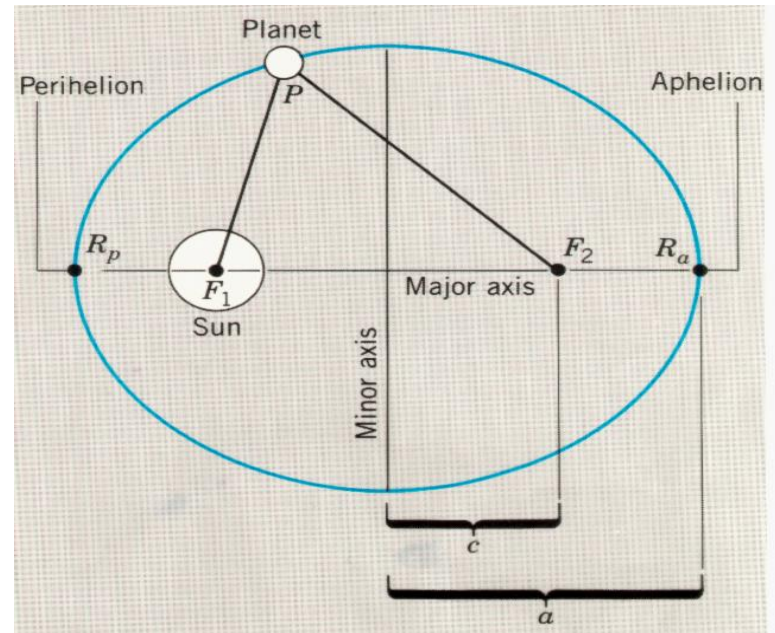
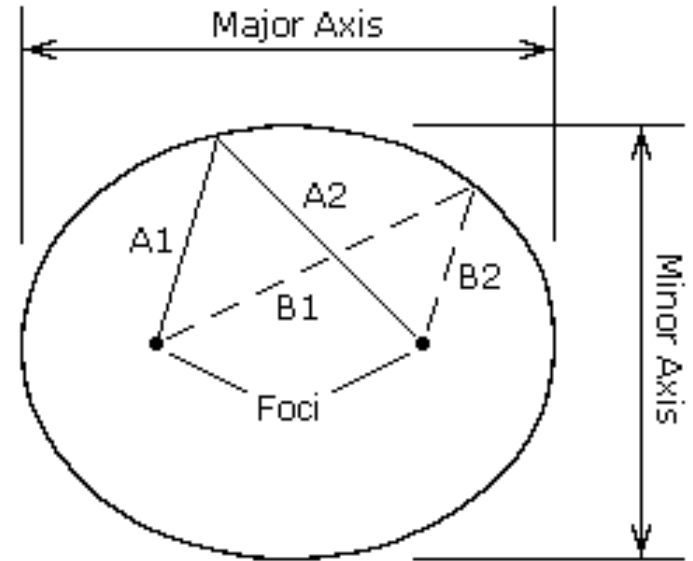
# Energy and Eccentricity

Conic Section	Eccentricity, $e$	Semi-major axis	Energy
Circle	0	= radius	$< 0$
Ellipse	$0 < e < 1$	$> 0$	$< 0$
Parabola	1	infinity	0
Hyperbola	$> 1$	$< 0$	$> 0$



# Major-Minor-Semimajor Axis

- The longest and shortest lines that can be drawn through the center of an ellipse are called the **major axis** and **minor axis**, respectively
- The ***semi-major axis*** ( $a$ ) is one-half of the major axis and represents a satellite's mean distance from its primary.  $2a$  is major axis.





# Semimajor Axis

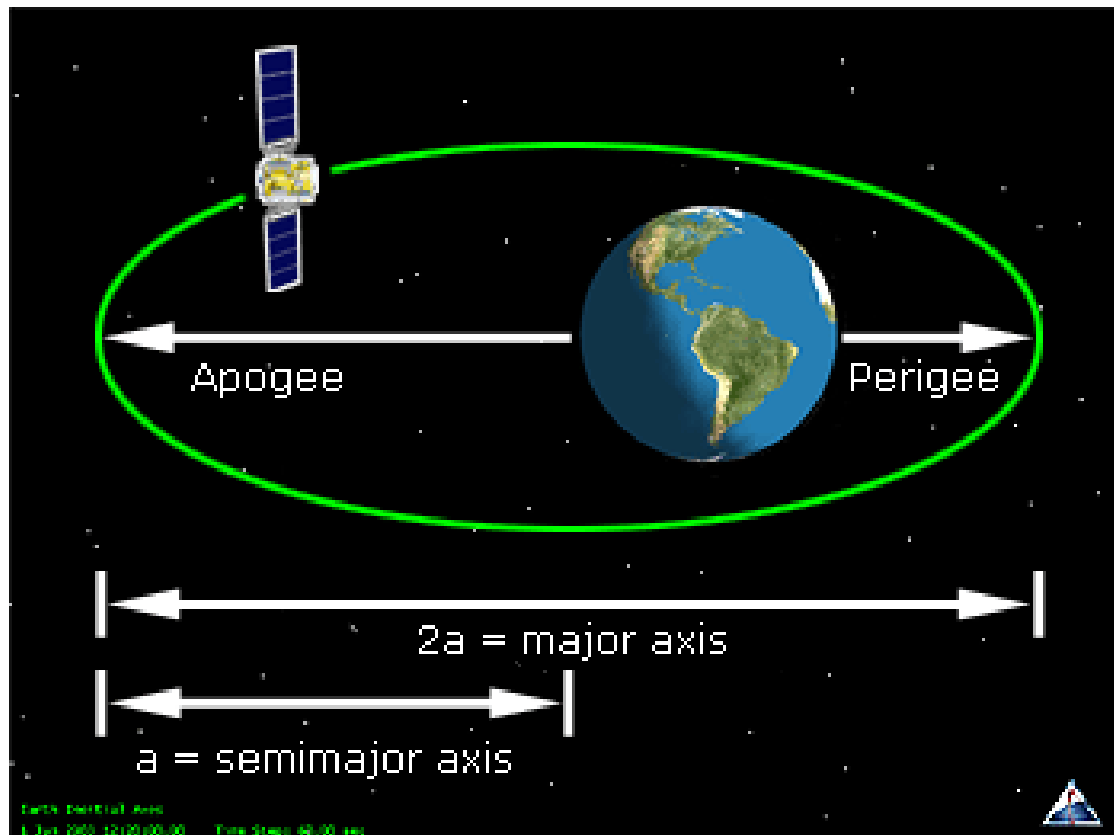
- We can express the semimajor axis in terms of the distance from the center of the Earth to apogee ( $R_{apogee}$ ) and perigee ( $R_{perigee}$ ). It expresses the size of the orbit. The semimajor axis can be found using:

$$a = \frac{R_{apogee} + R_{perigee}}{2}$$

- $a$  = semimajor axis ( $km$ )  
 $R_{apogee}$  = Distance from center of Earth to apogee ( $km$ )  
 $R_{perigee}$  = Distance from center of Earth to perigee ( $km$ )

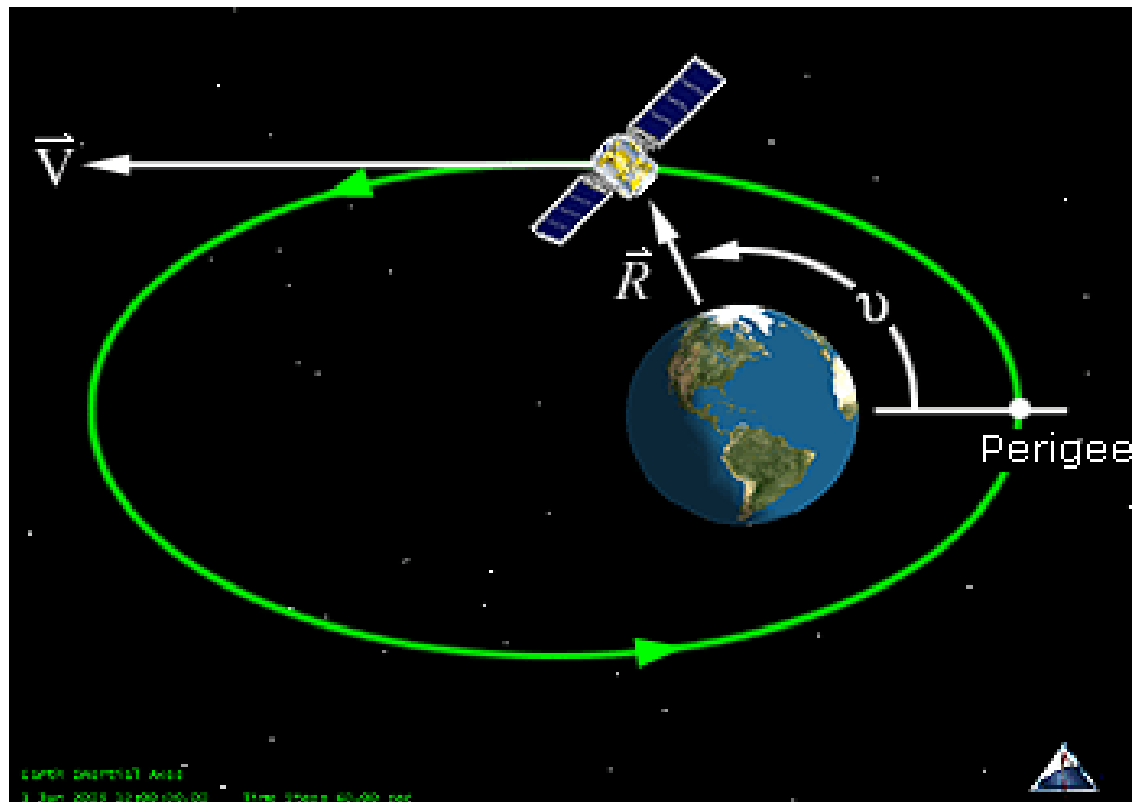
# Size of the Orbit

- Hence the semimajor axis actually tells us the size of the orbit.



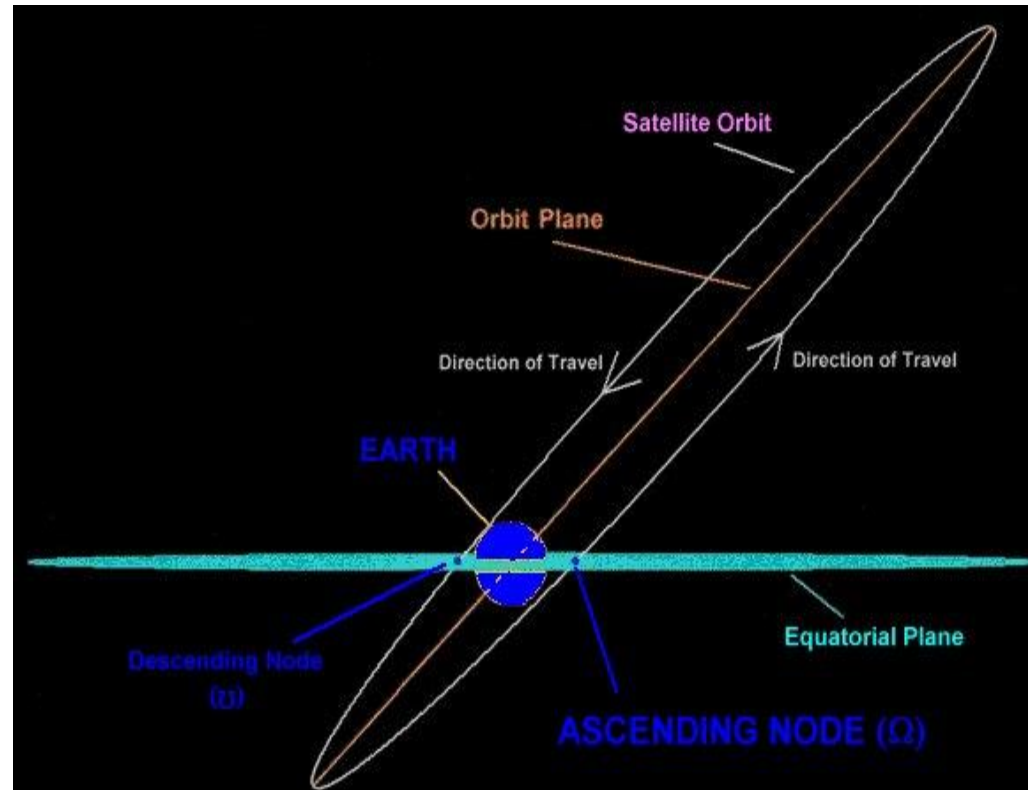
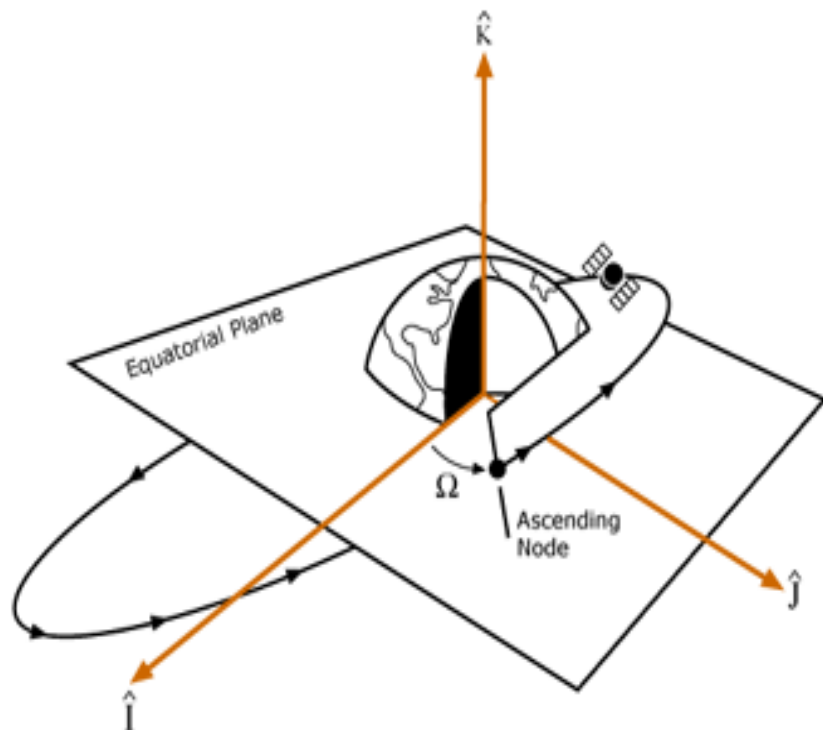
# True Anomaly

- It is the angle, measured positive in the direction of motion, between perigee and the satellite's position. It changes continuously during the orbit of the satellite.



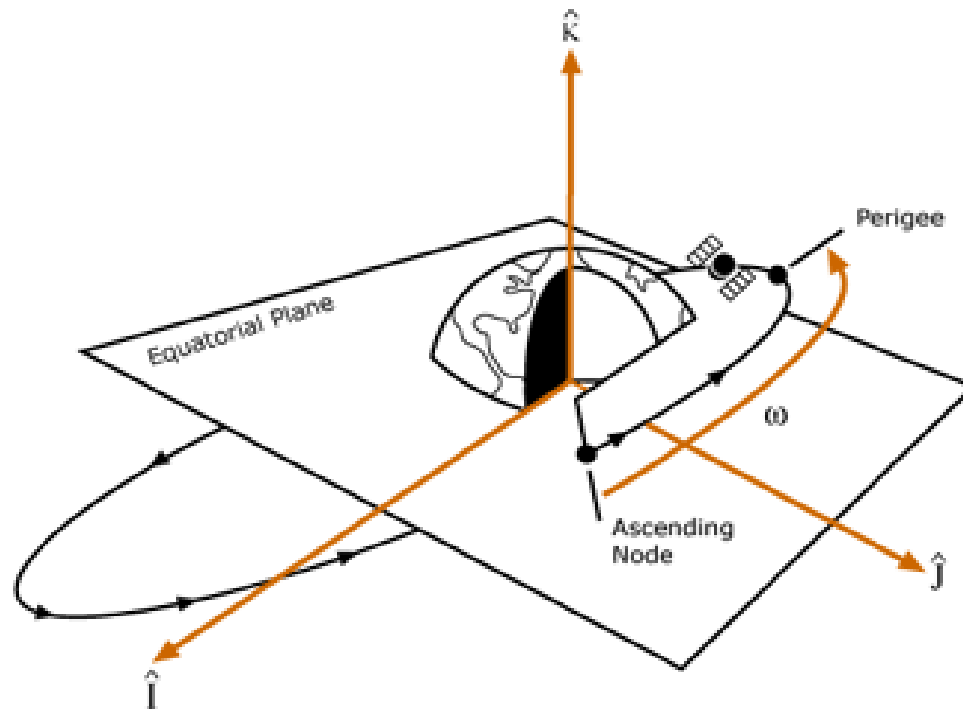
# Ascending Node

- We measure how an orbit is twisted by locating its *ascending node*, the point where the satellite crosses the equator moving south to north.



# Argument of Perigee

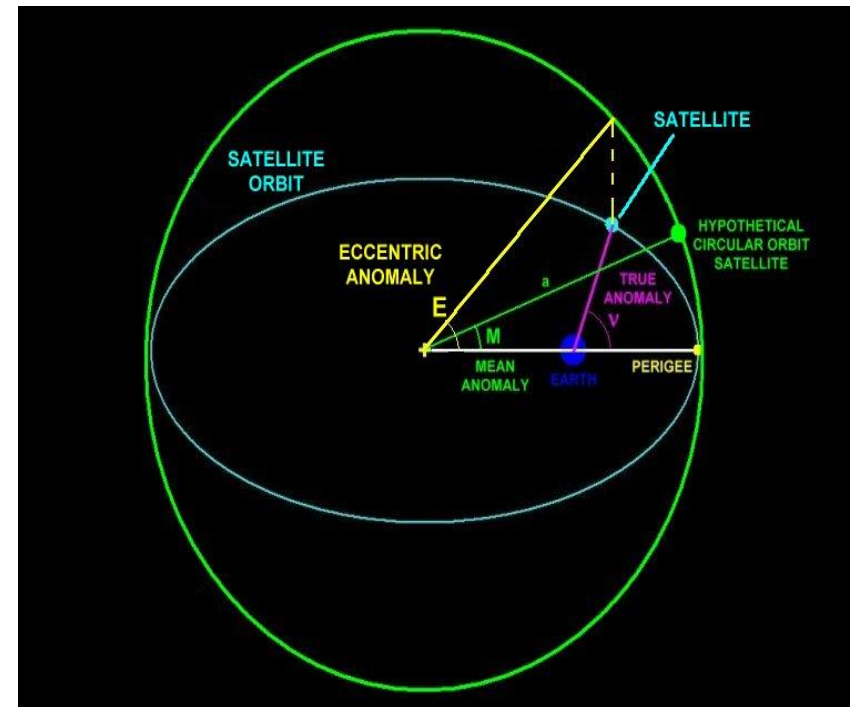
- The **argument of perigee**,  $\omega$  is the angular distance between the ascending node and perigee.



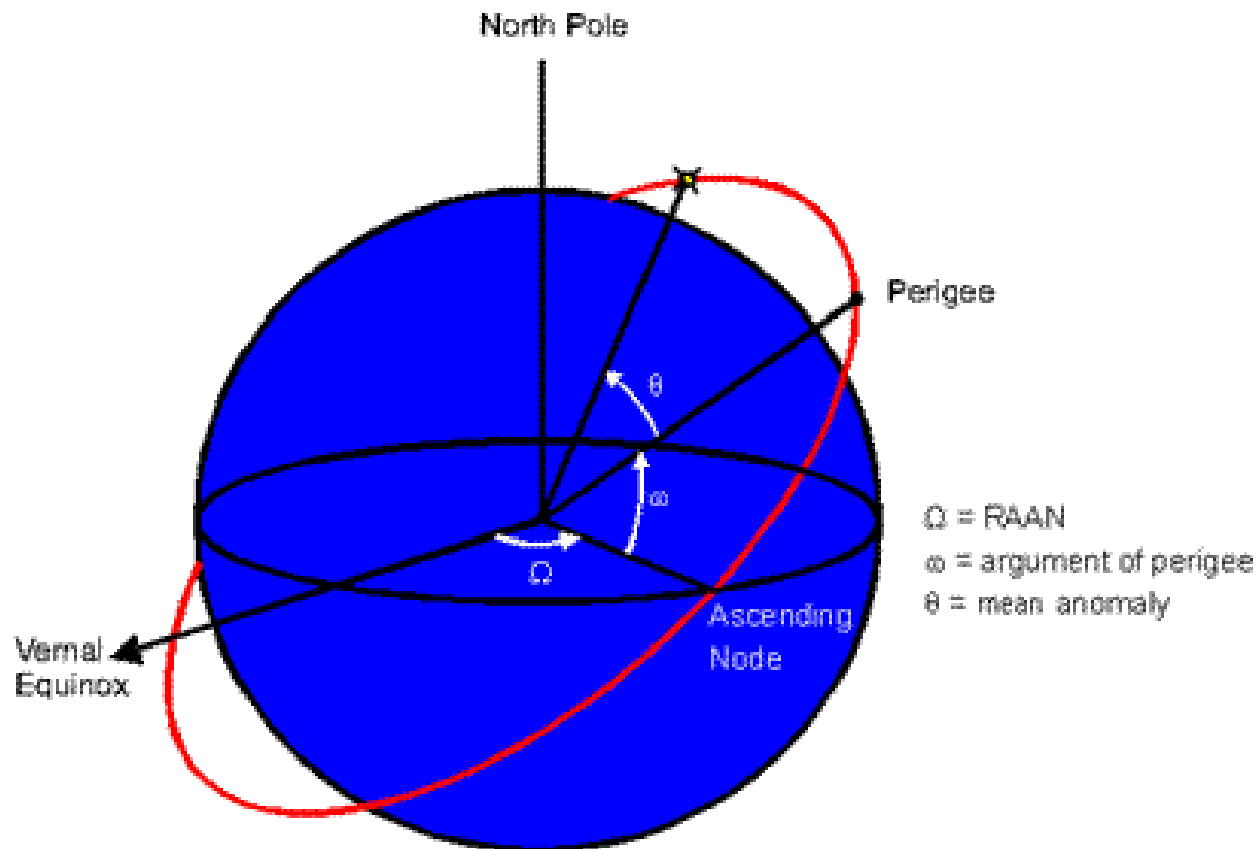
# Mean Anomaly and Eccentric Anomaly

## Anomaly

- **Mean Anomaly (M):** The angle measured since perigee that would be swept out by the satellite if its orbit were perfectly circular. The Mean Anomaly indicates where the satellite was in its orbit at a specific time.
- **Eccentric Anomaly (E):** The angle, measured since perigee, based on the hypothetical position on the circular orbit defined by a line perpendicular to the major axis that passes through the true position of the satellite and intersects with the circular orbit



# Orbital Parameters

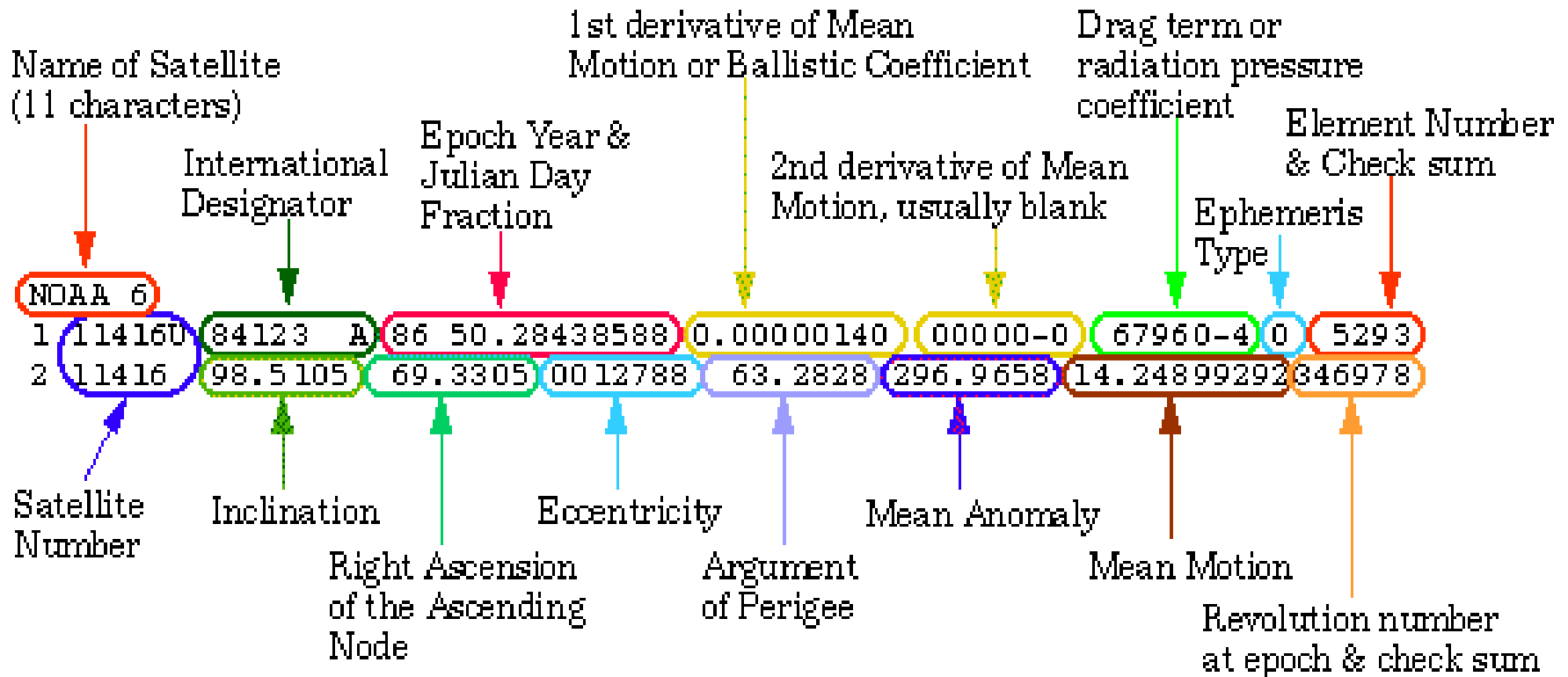


# Summary of Orbital Elements

Name	Symbol	Describes
Semimajor axis	$a$	Size (and energy)
Eccentricity	$e$	Shape ( $e = 0$ for circle, $0$ is less than $e$ is less than $1$ for ellipse, $e = 1$ for parabola, $e$ is greater than $1$ for hyperbola)
Inclination	$i$	Tilt of orbit plane with respect to the equator
Longitude of ascending node	$\Omega$	Twist of orbit with respect to the ascending node location
Argument of perigee	$\omega$	Location of perigee with respect to the ascending node
True anomaly	$\nu$	Location of satellite with respect to perigee



# Two Line Element Set Coordinate System



# Sample Orbital Information for a Satellite

- Sat Name: CARTOSAT-2
- CAT No. 37838
- DRAG 0.00001006
- BSTAR 14407-3
- Inclination 19.7947
- Right Ascension 62.7084
- Eccentricity 0.0060638
- Argument of perigee 17.8079
- Mean Anomaly 342.4581
- Mean Motion 14.2066792
- Element Set 2742
- Rev No. 25724 Epoch Time 11289.8779
- SemiMajor Axis 7201.23183
- Height above Equator 823.09
- Period ( in seconds ) 6081.646441
- Epoch Year -----> 2011
- Epoch Day of year-----> 289
- EpochTime -----> 21:04:10

# Sample Satellite Orbits

- The orbit of a satellite launched by the simple means of pushing it out of the bay of the Space Shuttle would have Orbital period 90 minutes, semi-major axis about 6500 km
- The motion of a spacecraft that is always located over the same part of the Earth would have Semi-major axis 22,000 miles (35,000 km), eccentricity 0

# Circular Orbital Equation

- Circular velocity of an orbit around an object is defined as:

$$V = \sqrt{\frac{k^2}{r}} = \sqrt{\frac{GM}{r}}$$

- Where for Earth

$$k^2 = GM = 3.98605 \times 10^{14} \text{ m}^3 / \text{s}^2$$

$$r = 6.378 \times 10^6 \text{ m}$$

Hence for escape from earth into circular orbit you would need a velocity of 7.9 km / sec

# Escape Orbital Equation

- For any vehicle to escape the Earth completely, it would need to have a parabolic or a hyperbolic trajectory.
- A parabolic / hyperbolic trajectory would have the least required potential and kinetic energy. Hence, the equation for parabolic orbital velocity will give the minimum escape velocity of 11.2 km/sec.

$$V = \sqrt{\frac{2k^2}{r}}$$

# Problem 1

- Calculate the velocity of an artificial satellite orbiting the Earth in a circular orbit at an altitude of 200 km above the Earth's surface.

- **ANSWER**

Radius of Earth = 6,378.14 km

GM of Earth =  $3.986005 \times 10^{14} \text{ m}^3/\text{s}^2$

Given:  $r = (6,378.14 + 200) \times 1,000 = 6,578,140 \text{ m}$

$v = \text{SQRT}[ \text{GM} / r ]$

$v = \text{SQRT}[ 3.986005 \times 10^{14} / 6,578,140 ]$

**$v = 7,784 \text{ m/s}$**

# Period Calculation of a Satellite

- The most simple equation for the period of a satellite is given by:

$$T = (2\pi r^{\frac{3}{2}}) / (k)$$

where  $k=1.9965 \times 10^7$

- The velocity of a satellite for circular orbit is:

$$\eta = \frac{2\pi}{T} = \frac{\sqrt{k}}{r^{\frac{3}{2}}}$$

## Problem 2

- Calculate the period of revolution for the satellite in problem 1

- **ANSWER**

Given:  $r = 6,578,140$  m

$$T = (2\pi r^{\frac{3}{2}}) / (k)$$

where  $k = 1.9965 \times 10^7$

$$T = 5,310 \text{ s}$$



# Problem 3

- Calculate the radius of orbit for a Earth satellite in a geosynchronous orbit, where the Earth's rotational period is 86,164.1 seconds.
- **ANSWER**

$$T = 86,164.1 \text{ s}$$

$$T = (2\pi r^{\frac{3}{2}}) / (k)$$

$$r = [ T^2 \times GM / (4 \times \pi^2) ]^{1/3}$$

$$r = [ 86,164.1^2 \times 3.986005 \times 10^{14} / (4 \times \pi^2) ]^{1/3}$$

$$r = 42,164,170 \text{ m}$$

# Velocity at Elliptical Orbit

$$V_p = \sqrt{\frac{2GMR_a}{R_p(R_a + R_p)}}$$

$$V_a = \sqrt{\frac{2GMR_p}{R_a(R_a + R_p)}}$$

# Problem 4

- An artificial Earth satellite is in an elliptical orbit which brings it to an altitude of 250 km at perigee and out to an altitude of 500 km at apogee. Calculate the velocity of the satellite at both perigee and apogee.

**ANSWER**

$$R_p = (6,378.14 + 250) \times 1,000 = 6,628,140 \text{ m}$$

$$R_a = (6,378.14 + 500) \times 1,000 = 6,878,140 \text{ m}$$

$$V_p = \text{SQRT}[ 2 \times 3.986005 \times 10^{14} \times 6,878,140 / (6,628,140 \times (6,878,140 + 6,628,140)) ]$$

$$V_p = 7,826 \text{ m/s}$$

$$V_p = \sqrt{\frac{2GMR_a}{R_p(R_a + R_p)}}$$

$$V_a = \text{SQRT}[ 2 \times 3.986005 \times 10^{14} \times 6,628,140 / (6,878,140 \times (6,878,140 + 6,628,140)) ]$$

$$V_a = 7,542 \text{ m/s}$$

$$V_a = \sqrt{\frac{2GMR_p}{R_a(R_a + R_p)}}$$

# Problem 5

- A satellite in Earth orbit passes through its perigee point at an altitude of 200 km above the Earth's surface and at a velocity of 7,850 m/s. Calculate the apogee altitude of the satellite.

- **ANSWER**

$$R_p = (6,378.14 + 200) \times 1,000 = 6,578,140 \text{ m}$$

$$V_p = 7,850 \text{ m/s}$$

Solve for  $R_a$  by equation:

$$V_p = \sqrt{\frac{2GM R_a}{R_p (R_a + R_p)}}$$

$$R_a = R_p / [2 \times GM / (R_p \times V_p^2) - 1]$$

$$R_a = 6,578,140 / [2 \times 3.986005 \times 10^{14} / (6,578,140 \times 7,850^2) - 1]$$

$$R_a = 6,805,140 \text{ m}$$

$$\text{Altitude @ apogee} = 6,805,140 / 1,000 - 6,378.14 = \mathbf{427.0 \text{ km}}$$

# Eccentricity of an Orbit

- Eccentricity of an orbit is given by the relation below as:

$$e = \frac{R_p V_p^2}{GM} - 1$$

# Problem 6

- Calculate the eccentricity of the orbit for the satellite in problem 5

- **ANSWER**

$R_p = 6,578,140$  m and  $V_p = 7,850$  m/s

With equation:

$$e = \frac{R_p V_p^2}{GM} - 1$$

$$e = R_p \times V_p^2 / GM - 1$$

$$e = 6,578,140 \times 7,850^2 / 3.986005 \times 10^{14} - 1$$

$$e = 0.01696$$

# Periapsis and Apoapsis Calculation

- If the semi-major axis  $a$  and the eccentricity  $e$  of an orbit are known, then the periapsis (perigee) and apoapsis (apogee) distances can be calculated by:

$$R_p = a(1 - e)$$

$$R_a = a(1 + e)$$

$$R_p + R_a = 2a$$

# Problem 7

- A satellite in Earth orbit has a semi-major axis of 6,700 km and an eccentricity of 0.01. Calculate the satellite's altitude at both perigee and apogee.

- **ANSWER**

$$a = 6,700 \text{ km and } e = 0.01$$

$$R_p = a \times (1 - e)$$

$$R_p = 6,700 \times (1 - .01)$$

$$R_p = 6,633 \text{ km}$$

$$\text{Altitude @ perigee} = 6,633 - 6,378.14 = 254.9 \text{ km}$$

$$R_a = a \times (1 + e)$$

$$R_a = 6,700 \times (1 + .01)$$

$$R_a = 6,767 \text{ km}$$

$$\text{Altitude @ apogee} = 6,767 - 6,378.14 = 388.9 \text{ km}$$



# Sample Orbit Determination

- If the space shuttle is in an altitude of 250 km in a circular orbit, then calculate the period of the orbit and its speed.
- The radius of the orbit= 6378.14 km + 250 =6628.14
- The period of the orbit is :

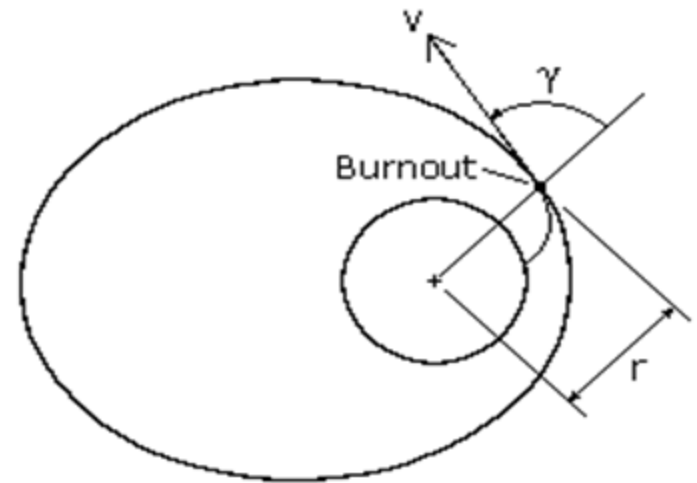
$$T = \frac{2\pi 6628.14^{3/2}}{k} = 5370.30s = 89 \text{ min } 30 \text{ sec}$$

- The velocity of the Shuttle is:

$$V = \sqrt{\frac{k^2}{r}} = \frac{3.956 \times 10^{14} \text{ m}^3 / \text{s}^2}{6.62814 \times 10^6} = 7.72 \text{ km/s}$$

# Orbit Determination

- A space vehicle's orbit may be determined from the position and the velocity of the vehicle at the beginning of its free flight. A vehicle's position and velocity can be described by the variables  $r$ ,  $v$ , and  $\gamma$ , where  $r$  is the vehicle's distance from the center of the Earth,  $v$  is its velocity, and  $\gamma$  is the angle between the position and the velocity vectors, called the **zenith angle**.
- If we let  $r_1$ ,  $v_1$ , and  $\gamma_1$  be the initial (launch) values of  $r$ ,  $v$ , and  $\gamma$ , then we may consider these as given quantities. If we let point  $P_2$  represent the perigee,



# Launch of a Space Vehicle

- As based upon the launch of a space vehicle, it is possible to determine its orbit parameters:

$$v_2 = V_p = \frac{r_1 v_1 \sin \gamma_1}{R_p}$$

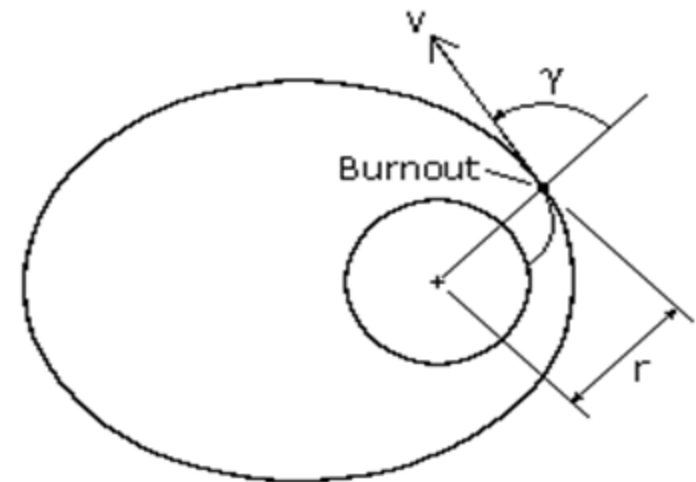
$$T_1 + V_1 = T_2 + V_2, \text{ or}$$

$$\frac{m v_1^2}{2} - \frac{GMm}{r_1} = \frac{m v_2^2}{2} - \frac{GMm}{r_2},$$

$$v_2^2 - v_1^2 = 2GM \left( \frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$\left( \frac{R_p}{r_1} \right)^2 (1 - C) + \left( \frac{R_p}{r_1} \right) C - \sin^2 \gamma_1 = 0$$

$$\text{where } C = \frac{2GM}{r_1 v_1^2}$$



# Eccentricity of an Orbit

- Eccentricity of an orbit by the initial launch parameters can be defined as:

$$e = \sqrt{\left(\frac{r_1 v_1^2}{GM} - 1\right)^2 \sin^2 \gamma_1 + \cos^2 \gamma_1}$$

# Problem 9

- Calculate the eccentricity of the orbit for the satellite with the following parameters?

Given:  $r_1 = 6,628,140$  m

$v_1 = 7,900$  m/s

$\gamma_1 = 89^\circ$

With the eccentricity equation:

$$e = \sqrt{\left(\frac{r_1 v_1^2}{GM} - 1\right)^2 \sin^2 \gamma_1 + \cos^2 \gamma_1}$$

$$e = \text{SQRT} \left[ \left( \frac{6,628,140 \times 7,900^2}{3.986005 \times 10^{14}} - 1 \right)^2 \times \sin^2(89) + \cos^2(89) \right]$$

$$e = 0.0416170$$

# True Anomaly

- To pin down a satellite's orbit in space, we need to know the angle,  $\nu$  the true anomaly, from the periapsis point to the launch point. This angle is given by:

$$\tan \nu = \frac{\left( \frac{r_1 v_1^2}{GM} \right) \sin \gamma_1 \cos \gamma_1}{\left( \frac{r_1 v_1^2}{GM} \right) \sin^2 \gamma_1 - 1}$$

# Problem 10

- A satellite is launched into Earth orbit where its launch vehicle burns out at an altitude of 250 km. At burnout the satellite's velocity is 7,900 m/s with the zenith angle equal to 89 degrees. Calculate the angle from perigee point to launch point for the satellite.

- **ANSWER**

$$r_1 = 6,628,140 \text{ m} \quad v_1 = 7,900 \text{ m/s} \quad \gamma_1 = 89^\circ$$

$$\tan \nu = \frac{\left( \frac{r_1 v_1^2}{GM} \right) \sin \gamma_1 \cos \gamma_1}{\left( \frac{r_1 v_1^2}{GM} \right) \sin^2 \gamma_1 - 1}$$

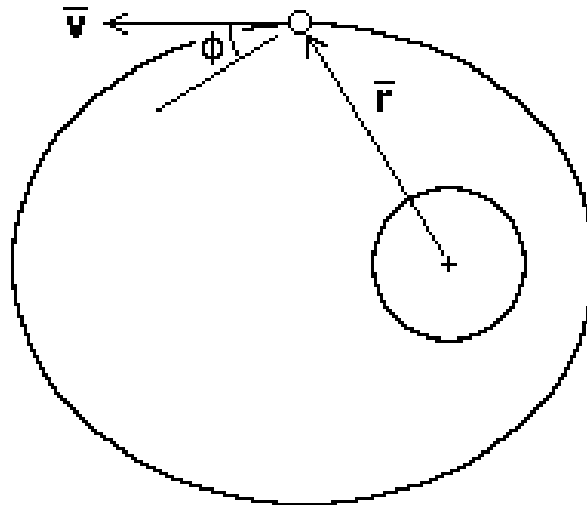
$$\tan \nu = (6,628,140 \times 7,900^2 / 3.986005 \times 10^{14}) \times \sin(89) \times \cos(89) / [(6,628,140 \times 7,900^2 / 3.986005 \times 10^{14}) \times \sin^2(89) - 1]$$

$$\tan \nu = 0.48329$$

$$\nu = \arctan(0.48329) = 25.794^\circ$$

# Flight Path Angle

- In most calculations, the complement of the zenith angle is used, denoted by  $\Phi$ . This angle is called the *flight-path angle*, and is positive when the velocity vector is directed away from the primary as shown:





# Eccentricity and True Anomaly by Flight Path Angle

$$e = \sqrt{\left(\frac{rv^2}{GM} - 1\right)^2 \cos^2 \phi + \sin^2 \phi}$$

$$\tan \nu = \frac{\left(\frac{rv^2}{GM}\right) \cos \phi \sin \phi}{\left(\frac{rv^2}{GM}\right) \cos^2 \phi - 1}$$

$$a = \frac{1}{\left(\frac{2}{r} - \frac{v^2}{GM}\right)}$$

# Problem 11

- A satellite is launched into Earth orbit where its launch vehicle burns out at an altitude of 250 km. At burnout the satellite's velocity is 7,900 m/s with the zenith angle equal to 89 degrees. Calculate the semi-major axis of the orbit for the satellite.

- **ANSWER**

$$a = \frac{1}{\left( \frac{2}{r} - \frac{v^2}{GM} \right)}$$

$$a = 1 / \left( 2 / 6,628,140 - 7,900^2 / 3.986005 \times 10^{14} \right)$$

$$a = 6,888,430 \text{ m}$$

# Locating the Satellite in Orbit

$$r_o = \frac{a(1 - e^2)}{1 + e \cos \nu}$$

Where:

$e$  : eccentricity of the orbit

$a$  : measure from the foci to the apogee

$r$  : radius from the foci of the planet

$\nu$  : True anomaly (measure of the angle from the perigee to the position of the satellite)

**The Rectangular Coordinates of a Satellite are:**

$$x_o = r_o \cos \phi_o$$

$$y_o = r_o \sin \phi_o$$

# Locating the Satellite in Orbit

- We can further calculate the flight path angle and the velocity of the spacecraft by the following relations:

$$\phi = \arctan\left(\frac{e \sin \nu}{1 + e \cos \nu}\right)$$

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

# Problem 12

- A satellite is in an orbit with a semi-major axis of 7,500 km and an eccentricity of 0.1. Calculate the length of its position vector, its flight-path angle, and its velocity when the satellite's true anomaly is 225 degrees.

- ANSWER**

Given:  $a = 7,500,000$  m       $e = 0.1$        $\nu = 225$  degrees

$$r_o = \frac{a(1-e^2)}{1+e \cos \nu} \quad \phi = \arctan\left(\frac{e \sin \nu}{1+e \cos \nu}\right) \quad v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$r = 7,500,000 \times (1 - 0.1^2) / (1 + 0.1 \times \cos(225))$$

$$r = 7,989,977 \text{ m}$$

$$\phi = \arctan[0.1 \times \sin(225) / (1 + 0.1 \times \cos(225))] =$$

$$\phi = -4.351 \text{ degrees}$$

$$v = \text{SQRT}[3.986005 \times 10^{14} \times (2 / 7,989,977 - 1 / 7,500,000)]$$

$$v = 6,828 \text{ m/s}$$

# Orbital Perturbations

- There are other forces acting on a satellite that perturb it away from the nominal orbit. These *perturbations*, or variations in the orbital elements, can be classified based on how they affect the Keplerian elements.
- *Secular variations* represent a linear variation in the element, *short-period variations* are periodic in the element with a period less than the orbital period, and *long-period variations* are those with a period greater than the orbital period. Because secular variations have long-term effects on orbit prediction (the orbital elements affected continue to increase or decrease)

# Third Body Perturbations

- The gravitational forces of the Sun and the Moon cause periodic variations in all of the orbital elements, but only the longitude of the ascending node, argument of perigee, and mean anomaly experience secular variations.
- These secular variations arise from a gyroscopic precession of the orbit about the ecliptic pole. The secular variation in mean anomaly is much smaller than the mean motion and has little effect on the orbit, however the secular variations in longitude of the ascending node and argument of perigee are important, especially for high-altitude orbits.

# Moon and Sun Perturbations

- For nearly circular orbits the equations for the secular rates of change resulting from the Sun and Moon are
- Longitude of the ascending node:

$$\dot{\Omega}_{\text{moon}} = -0.00338 \cos(i) / n$$

$$\dot{\Omega}_{\text{sun}} = -0.00154 \cos(i) / n$$

- Argument of perigee:

$$\dot{\omega}_{\text{moon}} = 0.00169(4 - 5 \sin^2 i) / n$$

$$\dot{\omega}_{\text{sun}} = 0.00077(4 - 5 \sin^2 i) / n$$

where  $i$  is the orbit inclination,  $n$  is the number of orbit revolutions per day,  $\dot{\Omega}$  and  $\dot{\omega}$  and are in degrees per day



# Problem 13

- Calculate the perturbations in longitude of the ascending node and argument of perigee caused by the Moon and Sun for the International Space Station orbiting at an altitude of 400 km, an inclination of 51.6 degrees, and with an orbital period of 92.6 minutes.
- $i = 51.6$  degrees       $n = 1436 / 92.6 = 15.5$  revolutions/day

$$\Omega_{\text{moon}} := -0.00338 \times \cos(51.6) / 15.5 = -0.000135 \text{ deg/day}$$

$$\Omega_{\text{sun}} = -0.00154 \times \cos(51.6) / 15.5 = -0.0000617 \text{ deg/day}$$

$$\omega_{\text{moon}} = 0.00169 \times (4 - 5 \times \sin^2 51.6) / 15.5 = 0.000101 \text{ deg/day}$$

$$\omega_{\text{sun}} = 0.00077 \times (4 - 5 \times \sin^2 51.6) / 15.5 = 0.000046 \text{ deg/day}$$

# Perturbations due to Non-Spherical Earth

- In fact, the Earth is neither homogeneous nor spherical. The most dominant features are a bulge at the equator, a slight pear shape, and flattening at the poles. For a potential function of the Earth, we can find a satellite's acceleration by taking the gradient of the potential function. The most widely used form of the geopotential function depends on latitude and geopotential coefficients,  $J_n$ , called the *zonal coefficients*.
- The potential generated by the non-spherical Earth causes periodic variations in all the orbital elements. The dominant effects, however, are secular variations in longitude of the ascending node and argument of perigee because of the Earth's oblateness, represented by the  $J_2$  term in the geopotential expansion.

# Perturbations due to Non-Spherical Earth

$$\Omega_{J_2} = -1.5nJ_2(R_E/a)^2(\cos i)(1-e^2)^{-2}$$

$$\approx -2.06474 \times 10^{14} a^{-7/2}(\cos i)(1-e^2)^{-2}$$

$$\omega_{J_2} = 0.75nJ_2(R_E/a)^2(4-5\sin^2 i)(1-e^2)^{-2}$$

$$\approx 1.03237 \times 10^{14} a^{-7/2}(4-5\sin^2 i)(1-e^2)^{-2}$$

- where  $n$  is the mean motion in degrees/day,  $J_2$  has the value 0.00108263,  $R_E$  is the Earth's equatorial radius,  $a$  is the semi-major axis in kilometers,  $i$  is the inclination,  $e$  is the eccentricity, and  $\Omega_{J_2}$  and  $\omega_{J_2}$  are in degrees/day. For satellites in GEO and below, the  $J_2$  perturbations dominate; for satellites above GEO the Sun and Moon perturbations dominate.
- *Molniya* orbits are designed so that the perturbations in argument of perigee are zero. This condition occurs when the term  $4-5\sin^2 i$  is equal to zero or, that is, when the inclination is either 63.4 or 116.6 degrees.

# Problem 14

- A satellite is in an orbit with a semi-major axis of 7,500 km, an inclination of 28.5 degrees, and an eccentricity of 0.1. Calculate the  $J_2$  perturbations in longitude of the ascending node and argument of perigee.

## ANSWER

Given:  $a = 7,500$  km    $i = 28.5$  degrees    $e = 0.1$

$$\begin{aligned} J_2 &= -2.06474 \times 10^{14} \times a^{-7/2} \times (\cos i) \times (1 - e^2)^{-2} \\ &= -2.06474 \times 10^{14} \times (7,500)^{-7/2} \times (\cos 28.5) \times (1 - (0.1)^2)^{-2} \\ &= -5.067 \text{ deg/day} \end{aligned}$$

$$\begin{aligned} J_2 &= 1.03237 \times 10^{14} \times a^{-7/2} \times (4 - 5 \times \sin^2 i) \times (1 - e^2)^{-2} \\ &= 1.03237 \times 10^{14} \times (7,500)^{-7/2} \times (4 - 5 \times \sin^2 28.5) \times (1 - (0.1)^2)^{-2} \\ &= 8.250 \text{ deg/day} \end{aligned}$$

# Perturbations from Atmospheric Drag

- Drag is the resistance offered by a gas or liquid to a body moving through it. A spacecraft is subjected to drag forces when moving through a planet's atmosphere. This drag is greatest during launch and reentry, however, even a space vehicle in low Earth orbit experiences some drag as it moves through the Earth's thin upper atmosphere. In time, the action of drag on a space vehicle will cause it to spiral back into the atmosphere, eventually to disintegrate or burn up. If a space vehicle comes within 120 to 160 km of the Earth's surface, atmospheric drag will bring it down in a few days, with final disintegration occurring at an altitude of about 80 km. Above approximately 600 km, on the other hand, drag is so weak that orbits usually last more than 10 years - beyond a satellite's operational lifetime. The deterioration of a spacecraft's orbit due to drag is called *decay*.

# Drag Force on a Body

- The drag force  $F_D$  on a body acts in the opposite direction of the velocity vector and is given by the equation:

$$F_D = \frac{1}{2} C_D \rho v^2 A$$

- where  $C_D$  is the drag coefficient,  $\rho$  is the air density,  $v$  is the body's velocity, and  $A$  is the area of the body normal to the flow. The drag coefficient is dependent on the geometric form of the body and is generally determined by experiment.
- Earth orbiting satellites typically have very high drag coefficients in the range of about 2 to 4.

# Satellite Decay Due to Drag

- The region above 90 km is the Earth's *thermosphere* where the absorption of extreme ultraviolet radiation from the Sun results in a very rapid increase in temperature with altitude. At approximately 200-250 km this temperature approaches a limiting value, the average value of which ranges between about 600 and 1,200 K over a typical solar cycle.
- Solar activity also has a significant affect on atmospheric density, with high solar activity resulting in high density. Below about 150 km the density is not strongly affected by solar activity; however, at satellite altitudes in the range of 500 to 800 km, the density variations between solar maximum and solar minimum are approximately two orders of magnitude.
- The large variations imply that satellites will decay more rapidly during periods of solar maxima and much more slowly during solar minima.

# Decay Analysis for Circular Orbits

- For circular orbits we can approximate the changes in semi-major axis, period, and velocity per revolution using the following equations:

$$\Delta a_{\text{rev}} = \frac{-2\pi C_D A P a^2}{m}$$

$$\Delta P_{\text{rev}} = \frac{-6\pi^2 C_D A P a^2}{mV}$$

$$\Delta V_{\text{rev}} = \frac{\pi C_D A P a V}{m}$$

- where  $a$  is the semi-major axis,  $P$  is the orbit period, and  $V$ ,  $A$  and  $m$  are the satellite's velocity, area, and mass respectively. The term  $m/(C_D A)$ , called the *ballistic coefficient*, is given as a constant for most satellites. Drag effects are strongest for satellites with low ballistic coefficients, this is, light vehicles with large frontal areas.



# Satellite Lifetime Due to Drag

- A rough estimate of a satellite's lifetime,  $L$ , due to drag can be computed from:

$$L \approx \frac{-H}{\Delta a_{\text{rev}}}$$

- where  $H$  is the atmospheric density scale height.

# Atmospheric Properties

Physical Properties of U.S. Standard Atmosphere, 1976 in SI Units

Altitude (meters)	Temperature (K)	Pressure (Pa)	Density (kg/m <sup>3</sup> )	Viscosity (N-s/m <sup>2</sup> )
-2,000	301.15	1.27774E+5	1.47808	1.87630E-5
-1,000	294.65	1.13929E+5	1.34700	1.84434E-5
0	288.15	1.01325E+5	1.22500	1.81206E-5
1,000	281.65	8.98746E+4	1.11164	1.77943E-5
2,000	275.15	7.94952E+4	1.00649	1.74645E-5
3,000	268.65	7.01085E+4	9.09122E-1	1.71311E-5
4,000	262.15	6.16402E+4	8.19129E-1	1.67940E-5
5,000	255.65	5.40199E+4	7.36116E-1	1.64531E-5
6,000	249.15	4.71810E+4	6.59697E-1	1.61084E-5
7,000	242.65	4.10607E+4	5.89501E-1	1.57596E-5
8,000	236.15	3.55998E+4	5.25168E-1	1.54068E-5
9,000	229.65	3.07425E+4	4.66348E-1	1.50498E-5
10,000	223.15	2.64363E+4	4.12707E-1	1.46884E-5
12,000	216.65	1.93304E+4	3.10828E-1	1.43226E-5
15,000	216.65	1.20446E+4	1.93674E-1	1.43226E-5
20,000	216.65	5.47489E+3	8.80349E-2	1.43226E-5
25,000	221.65	2.51102E+3	3.94658E-2	1.46044E-5
30,000	226.65	1.17187E+3	1.80119E-2	1.48835E-5
35,000	237.05	5.58924E+2	8.21392E-3	1.54559E-5
40,000	251.05	2.77522E+2	3.85101E-3	1.62096E-5
45,000	265.05	1.43135E+2	1.88129E-3	1.69449E-5
50,000	270.65	7.59448E+1	9.77525E-4	1.72341E-5
60,000	245.45	2.03143E+1	2.88321E-4	1.59104E-5
70,000	217.45	4.63422	7.42430E-5	1.43679E-5
80,000	196.65	8.86280E-1	1.57005E-5	1.31682E-5
84,852	186.95	3.73384E-1	6.95788E-6	1.25915E-5

## Basic Assumptions

Air is a clean, dry, perfect gas mixture; Specific heat ratio = 1.40; Molecular weight to 86 km = 28.9644

Principal sea-level constituents: N<sub>2</sub>-78.084%, O<sub>2</sub>-20.9476%, Ar-0.934%, CO<sub>2</sub>-0.0314%, Ne-0.001818%, He-0.000524%, CH<sub>4</sub>-0.0002%

## Problem 15

A satellite is in a circular Earth orbit at an altitude of 400 km. The satellite has a cylindrical shape 2 m in diameter by 4 m long and has a mass of 1,000 kg. The satellite is traveling with its long axis perpendicular to the velocity vector and its drag coefficient is 2.67. Calculate the perturbations due to atmospheric drag and estimate the satellite's lifetime.

### SOLUTION,

Given:

$$a = (6,378.14 + 400) \times 1,000 = \underline{6,778,140 \text{ m}}$$

$$A = 2 \times 4 = 8 \text{ m}^2$$

$$m = 1,000 \text{ kg}$$

$$C_D = 2.67$$

- From [Atmosphere Properties](#),

$$\rho = 2.62 \times 10^{-12} \text{ kg/m}^3$$

$$H = 58.2 \text{ km}$$

- $V = \text{SQRT}[ GM / a ]$

$$V = \text{SQRT}[ 3.986005 \times 10^{14} / 6,778,140 ]$$

$$V = \underline{7,669 \text{ m/s}}$$

Use Equations given for decay analysis in circular orbit

$$\Delta a_{\text{rev}} = (-2 \times \pi \times C_D \times A \times \rho \times a^2) / m$$

$$\Delta a_{\text{rev}} = (-2 \times \pi \times 2.67 \times 8 \times 2.62 \times 10^{-12} \times 6,778,140^2) / 1,000$$

$$\Delta a_{\text{rev}} = \underline{-16.2 \text{ m}}$$

$$\Delta P_{\text{rev}} = (-6 \times \pi^2 \times C_D \times A \times \rho \times a^2) / (m \times V)$$

$$\Delta P_{\text{rev}} = (-6 \times \pi^2 \times 2.67 \times 8 \times 2.62 \times 10^{-12} \times 6,778,140^2) / (1,000 \times 7,669)$$

$$\Delta P_{\text{rev}} = \underline{-0.0199 \text{ s}}$$

$$\Delta V_{\text{rev}} = (\pi \times C_D \times A \times \rho \times a \times V) / m$$

$$\Delta V_{\text{rev}} = (\pi \times 2.67 \times 8 \times 2.62 \times 10^{-12} \times 6,778,140 \times 7,669) / 1,000$$

$$\Delta V_{\text{rev}} = \underline{0.00914 \text{ m/s}}$$

Equation (4.56),

$$L \sim -H / \Delta a_{\text{rev}}$$

$$L \sim -(58.2 \times 1,000) / -16.2 \quad (\times 1000 \text{ because of meter conversion})$$

$$L \sim \underline{3,600 \text{ revolutions}}$$

# Perturbations from Solar Radiation

- Solar radiation pressure causes periodic variations in all of the orbital elements. The magnitude of the acceleration in  $\text{m/s}^2$  arising from solar radiation pressure is:

$$a_R = \frac{-4.5 \times 10^{-8} A}{m}$$

- where  $A$  is the cross-sectional area of the satellite exposed to the Sun and  $m$  is the mass of the satellite in kilograms.
- For satellites below 800 km altitude, acceleration from atmospheric drag is greater than that from solar radiation pressure; above 800 km, acceleration from solar radiation pressure is greater.