Satellite Orbital Maneuvers and Transfers

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Orbit Maneuvers

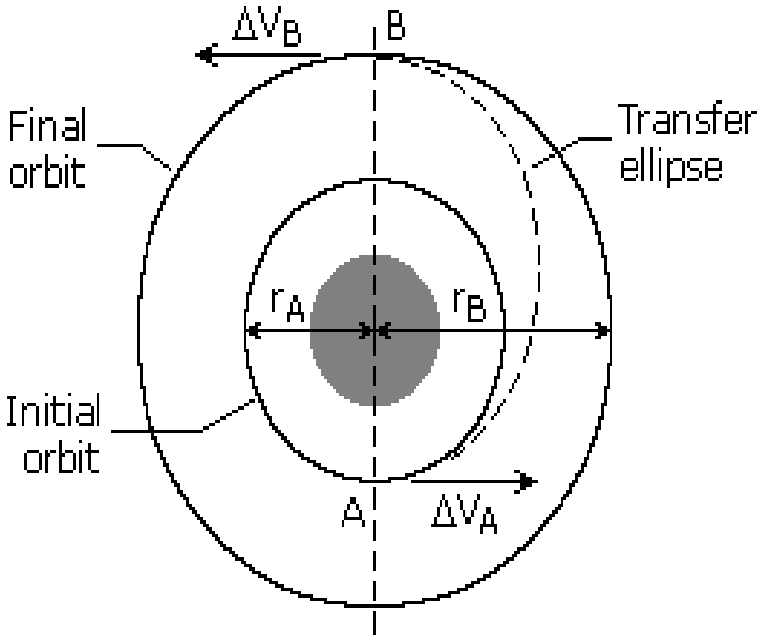
- At some point during the lifetime of most space vehicles or satellites, we must change one or more of the orbital elements. For example, we may need to transfer from an initial parking orbit to the final mission orbit, rendezvous with or intercept another spacecraft, or correct the orbital elements to adjust for the perturbations discussed in the previous section. Most frequently, we must change the orbit altitude, plane, or both. To change the orbit of a space vehicle, we have to change its velocity vector in magnitude or direction.

Orbit Maneuvers Strategy

- Most propulsion systems operate for only a short time compared to the orbital period, thus we can treat the maneuver as an impulsive change in velocity while the position remains fixed.
- For this reason, any maneuver changing the orbit of a space vehicle must occur at a point where the old orbit intersects the new orbit.
- If the orbits do not intersect, we must use an intermediate orbit that intersects both. In this case, the total maneuver will require at least two propulsive burns.

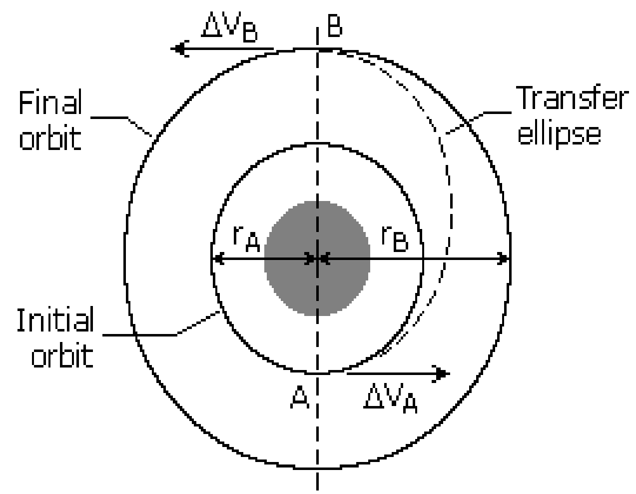
Orbit Attitude Transfer

- The most common type of in-plane maneuver changes the size and energy of an orbit, usually from a low-altitude parking orbit to a higher-altitude mission orbit such as a geosynchronous orbit. Because the initial and final orbits do not intersect, the maneuver requires a transfer orbit. Figure represents a *Hohmann transfer orbit*.
- In this case, the transfer orbit's ellipse is tangent to both the initial and final orbits at the transfer orbit's perigee and apogee respectively. The orbits are tangential, so the velocity vectors are collinear, and the Hohmann transfer represents the most fuel-efficient transfer between two circular, coplanar orbits.
- When transferring from a smaller orbit to a larger orbit, the change in velocity is applied in the direction of motion; when transferring from a larger orbit to a smaller, the change of velocity is opposite to the direction of motion.



Hohmann Transfer

- The total change in velocity required for the orbit transfer is the sum of the velocity changes at perigee and apogee of the transfer ellipse. Since the velocity vectors are collinear, the velocity changes are just the differences in magnitudes of the velocities in each orbit. If we know the initial and final orbits, r_A and r_B, we can calculate the total velocity change using the following equations

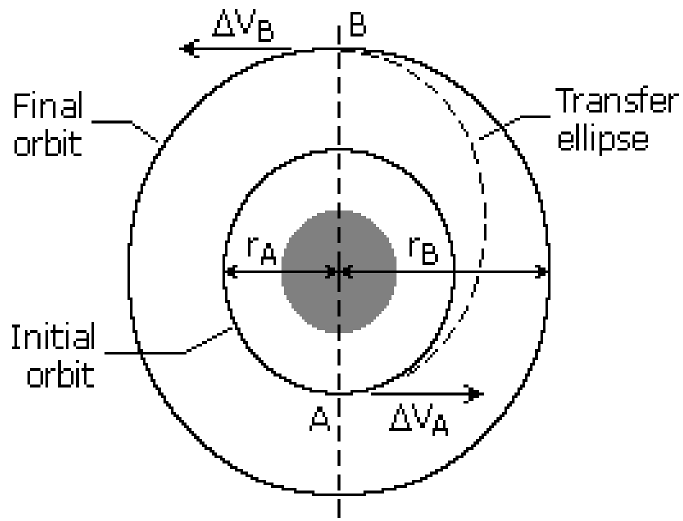


$$a_{tx} = \frac{r_A + r_B}{2}$$ semi-major axis of transfer ellipse

$$V_{iA} = \sqrt{\frac{GM}{r_A}}$$ initial velocity at point A

$$V_{fB} = \sqrt{\frac{GM}{r_B}}$$ final velocity at point B

Hohmann Transfer Equations



$$V_{txA} = \sqrt{GM\left(\frac{2}{r_A} - \frac{1}{a_{tx}}\right)}$$ velocity on transfer orbit at inital orbit (point A)

$$V_{txB} = \sqrt{GM\left(\frac{2}{r_B} - \frac{1}{a_{tx}}\right)}$$ velocity on transfer orbit at final orbit (point B)

$\Delta V_A = V_{txA} - V_{iA}$ initial velocity change

$\Delta V_B = V_{fB} - V_{txB}$ final velocity change

$\Delta V_T = \Delta V_A + \Delta V_B$ total velocity change

Problem 1

A spacecraft is in a circular parking orbit with an altitude of 200 km. Calculate the velocity change required to perform a Hohmann transfer to a circular orbit at geosynchronous altitude.

Given: $r_A = (6{,}378.14 + 200) \times 1{,}000 = \underline{6{,}578{,}140 \text{ m}}$

For geosynchronous orbits, $r_B = 42{,}164{,}170$ m

Equations given previously:

$a_{tx} = (r_A + r_B) / 2$

$a_{tx} = (6{,}578{,}140 + 42{,}164{,}170) / 2$

$a_{tx} = \underline{24{,}371{,}155 \text{ m}}$

$Vi_A = \text{SQRT}[\, GM / r_A \,]$

$Vi_A = \text{SQRT}[\, 3.986005 \times 10^{14} / 6{,}578{,}140 \,]$

$Vi_A = \underline{7{,}784 \text{ m/s}}$

$Vf_B = \text{SQRT}[\, GM / r_B \,]$

$Vf_B = \text{SQRT}[\, 3.986005 \times 10^{14} / 42{,}164{,}170 \,]$

$Vf_B = \underline{3{,}075 \text{ m/s}}$

$Vtx_A = \text{SQRT}[\, GM \times (2 / r_A - 1 / a_{tx})]$

$Vtx_A = \text{SQRT}[\, 3.986005 \times 10^{14} \times (2 / 6{,}578{,}140 - 1 / 24{,}371{,}155)]$

$Vtx_A = \underline{10{,}239 \text{ m/s}}$

$Vtx_B = SQRT[\ GM \times (2 / r_B - 1 / a_{tx})]$

$Vtx_B = SQRT[\ 3.986005 \times 10^{14} \times (2 / 42{,}164{,}170 - 1 / 24{,}371{,}155)]$

$Vtx_B =$ <u>1,597 m/s</u>

$\Delta V_A = Vtx_A - Vi_A$

$\Delta V_A = 10{,}239 - 7{,}784$

$\Delta V_A =$ <u>2,455 m/s</u>

$\Delta V_B = Vf_B - Vtx_B$

$\Delta V_B = 3{,}075 - 1{,}597$

$\Delta V_B =$ <u>1,478 m/s</u>

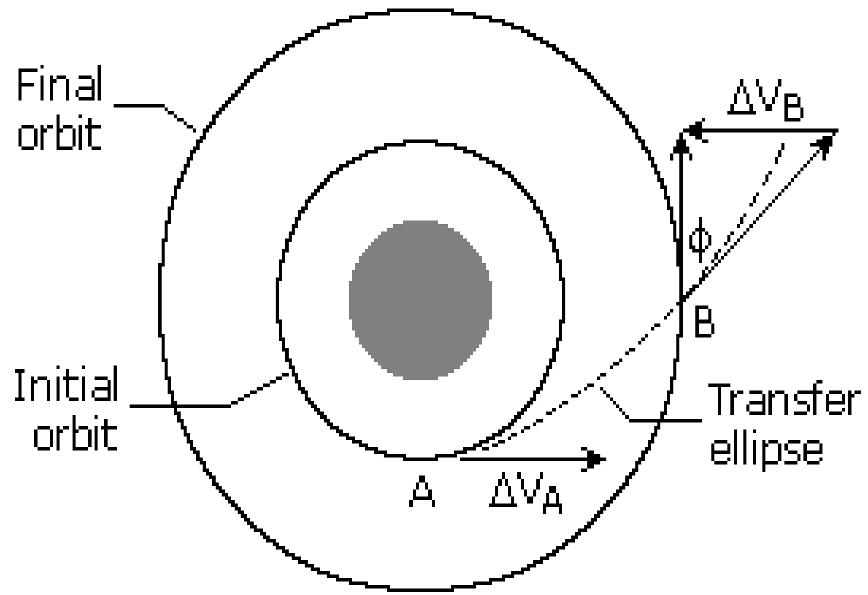
$\Delta V_T = \Delta\ V_A + \Delta\ V_B$

$\Delta V_T = 2{,}455 + 1{,}478$

$\Delta V_T =$ <u>3,933 m/s</u>

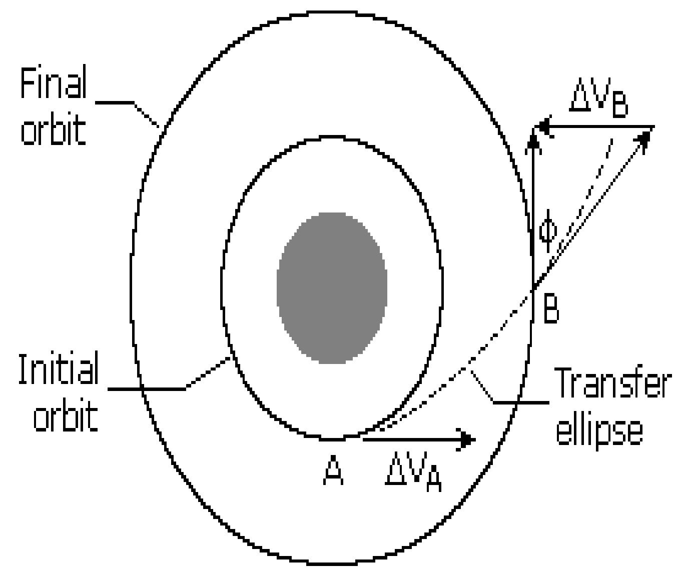
One Tangent Burn

- Ordinarily we want to transfer a space vehicle using the smallest amount of energy, which usually leads to using a Hohmann transfer orbit. However, sometimes we may need to transfer a satellite between orbits in less time than that required to complete the Hohmann transfer. Figure shows a faster transfer called the ***One-Tangent Burn***. In this instance the transfer orbit is tangential to the initial orbit. It intersects the final orbit at an angle equal to the flight path angle of the transfer orbit at the point of intersection.

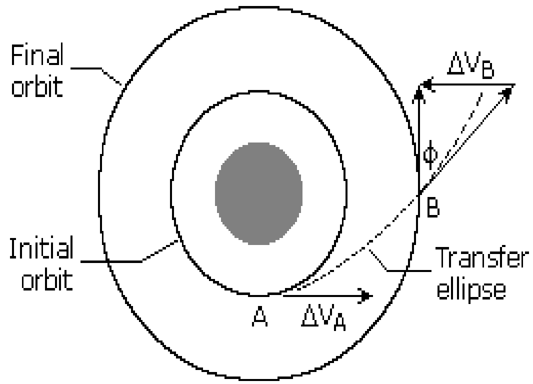


One Tangent Burn Maneuver

- An infinite number of transfer orbits are tangential to the initial orbit and intersect the final orbit at some angle. Thus, we may choose the transfer orbit by specifying the size of the transfer orbit, the angular change of the transfer, or the time required to complete the transfer. We can then define the transfer orbit and calculate the required velocities.
- For example, we may specify the size of the transfer orbit, choosing any semi-major axis that is greater than the semi-major axis of the Hohmann transfer ellipse. Once we know the semi-major axis of the ellipse, a_{tx}, we can calculate the eccentricity, angular distance traveled in the transfer, the velocity change required for the transfer, and the time required to complete the transfer.



One Tangent Burn Equations



$$e = 1 - \frac{r_A}{a_{tx}}$$ eccentricity of transfer ellipse

$$\nu = \arccos\left[\frac{\left(\frac{a_{tx}(1-e^2)}{r_B} - 1\right)}{e}\right]$$ true anomaly at second burn

$$\phi = \arctan\left(\frac{e \sin\nu}{1 + e\cos\nu}\right)$$ flight-path angle at second burn

$$\Delta V_B = \sqrt{V_{txB}^2 + V_{fB}^2 - 2 V_{txB} V_{fB} \cos\phi}$$ final velocity change

$$E = \arctan\left(\frac{\sqrt{1-e^2}\sin\nu}{e + \cos\nu}\right)$$ eccentric anomaly

$$TOF = (E - e\sin E)\sqrt{\frac{a^3}{GM}}$$ time-of-flight, E in radians

PROBLEM 2: A satellite is in a circular parking orbit with an altitude of 200 km. Using a one-tangent burn, it is to be transferred to geosynchronous altitude using a transfer ellipse with a semi-major axis of 30,000 km. Calculate the total required velocity change and the time required to complete the transfer.

SOLUTION,

Given: r_A = (6,378.14 + 200) × 1,000 = 6,578,140 m

r_B = 42,164,170 m

a_{tx} = 30,000 × 1,000 = 30,000,000 m ;

Equations for One Tangent Burn

- **$e = 1 - r_A / a_{tx}$**

e = 1 - 6,578,140 / 30,000,000

e = 0.780729

- **ν= arccos[(a_{tx} × (1 - e2) / r_B - 1) / e]**

ν = arccos[(30,000,000 × (1 - 0.780729^2) / 42,164,170 - 1) / 0.780729]

ν = 157.670 degrees

- **ϕ= arctan[e × sin ν / (1 + e × cos ν)]**

ϕ = arctan[0.780729 × sin(157.670) / (1 + 0.780729 × cos(157.670))]

ϕ = 46.876 degrees ;

Equations from Hohmann Transfer

- **ViA = SQRT[GM / rA]**

ViA = SQRT[3.986005×10^14 / 6,578,140]

ViA = 7,784 m/s

- **VfB = SQRT[GM / rB]**

VfB = SQRT[3.986005×10^14 / 42,164,170]

VfB = 3,075 m/s

- $\mathbf{Vtx_A = SQRT[\ GM \times (2 / r_A - 1 / a_{tx})]}$

$Vtx_A = SQRT[\ 3.986005 \times 10^{14} \times (2 / 6{,}578{,}140 - 1 / 30{,}000{,}000)]$

$Vtx_A =$ 10,388 m/s

- $Vtx_B = SQRT[\ GM \times (2 / r_B - 1 / a_{tx})]$

$Vtx_B = SQRT[\ 3.986005 \times 10^{14} \times (2 / 42{,}164{,}170 - 1 / 30{,}000{,}000)]$

$Vtx_B =$ 2,371 m/s

- $\Delta V_A = Vtx_A - Vi_A$

$\Delta\ V_A = 10{,}388 - 7{,}784$

$\Delta\ V_A =$ 2,604 m/s

- $\mathbf{\Delta\ V_B = SQRT[\ Vtx_B{}^2 + Vf_B{}^2 - 2 \times Vtx_B \times Vf_B \times \cos\phi\ ]}$

$\Delta\ V_B = SQRT[\ 2{,}3712 + 3{,}0752 - 2 \times 2{,}371 \times 3{,}075 \times \cos(46.876)]$

$\Delta\ V_B =$ 2,260 m/s

- **Δ VT = Δ VA + Δ VB**

Δ VT = 2,604 + 2,260

Δ VT = 4,864 m/s

- $\mathbf{E = \arctan[(1 - e^2)^{1/2} \times \sin \nu / (e + \cos \nu)]}$

$E = \arctan[(1 - 0.780729^2)^{1/2} \times \sin(157.670) / (0.780729 + \cos(157.670))]$

E = 2.11688 radians

- $\mathbf{TOF = (E - e \times \sin E) \times SQRT[\ a_{tx}{}^3 / GM\ ]}$

$TOF = (2.11688 - 0.780729 \times \sin(2.11688)) \times SQRT[\ 30{,}000{,}000^3 / 3.986005 \times 10^{14}\ ]$

TOF = 11,931 s = 3.314 hours

Spiral Transfer

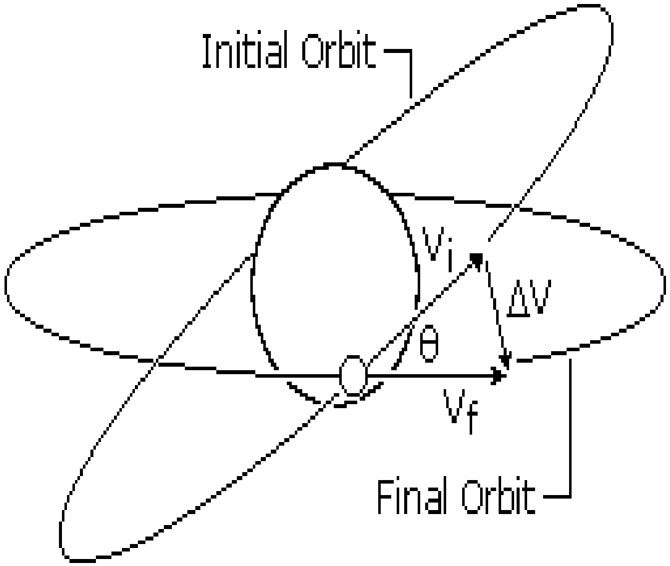
- Another option for changing the size of an orbit is to use electric propulsion to produce a constant low-thrust burn, which results in a *spiral transfer*. We can approximate the velocity change for this type of orbit transfer by

$$\Delta V = V_2 - V_1$$

- where the velocities are the circular velocities of the two orbits.

Orbit Plane Changes

- To change the orientation of a satellite's orbital plane, typically the inclination, we must change the direction of the velocity vector. This maneuver requires a component of *V* to be perpendicular to the orbital plane and, therefore, perpendicular to the initial velocity vector. If the size of the orbit remains constant, the maneuver is called a *simple plane change*. We can find the required change in velocity by using the law of cosines. For the case in which V_f is equal to V_i, this expression reduces to



$$\Delta V = 2 V_i \sin\left(\frac{\theta}{2}\right)$$

- where V_i is the velocity before and after the burn, and θ is the angle change required.

PROBLEM 3

Calculate the velocity change required to transfer a satellite from a circular 600 km orbit with an inclination of 28 degrees to an orbit of equal size with an inclination of 20 degrees.

SOLUTION,

Given: r = (6,378.14 + 600) × 1,000 = 6,978,140 m

θ= 28 - 20 = 8 degrees

Vi = SQRT[GM / r]

Vi = SQRT[3.986005×10^{14} / 6,978,140]

Vi = 7,558 m/s

$$\Delta V = 2V_i \sin\left(\frac{\theta}{2}\right)$$

ΔV = 2 × Vi × sin(θ /2)

Δ V = 2 × 7,558 × sin(8/2)

Δ V = 1,054 m/s

Location of Orbit Plane Changes

- Plane changes are very expensive in terms of the required change in velocity and resulting propellant consumption. To minimize this, we should change the plane at a point where the velocity of the satellite is a minimum: at apogee for an elliptical orbit. In some cases, it may even be cheaper to boost the satellite into a higher orbit, change the orbit plane at apogee, and return the satellite to its original orbit.
- Typically, orbital transfers require changes in both the size and the plane of the orbit, *such as transferring from an inclined parking orbit at low altitude to a zero-inclination orbit at geosynchronous altitude*.
- We can do this transfer in two steps: a Hohmann transfer to change the size of the orbit and a simple plane change to make the orbit equatorial. A more efficient method (less total change in velocity) would be to combine the plane change with the tangential burn at apogee of the transfer orbit.

Orbit Plane Change Equation

- As we must change both the magnitude and direction of the velocity vector, we can find the required change in velocity using the law of cosines:

$$\Delta V = \sqrt{V_i^2 + V_f^2 - 2 V_i V_f \cos\theta}$$

- where V_i is the initial velocity, V_f is the final velocity, and θ is the angle change required. As can be seen from equation, a small plane change can be combined with an altitude change for almost no cost in delta *V* or propellant. Consequently, in practice, geosynchronous transfer is done with a small plane change at perigee and most of the plane change at apogee.

PROBLEM 4

A satellite is in a parking orbit with an altitude of 200 km and an inclination of 28 degrees. Calculate the total velocity change required to transfer the satellite to a zero-inclination geosynchronous orbit using a Hohmann transfer with a combined plane change at apogee.

Given: $r_A = (6{,}378.14 + 200) \times 1{,}000 = 6{,}578{,}140$ m

$r_B = 42{,}164{,}170$ m

$\theta = 28$ degrees

From problem 1,

$Vf_B = 3{,}075$ m/s

$Vtx_B = 1{,}597$ m/s

$\Delta V_A = 2{,}455$ m/s

Equation (4.74),

$$\mathbf{\Delta V_B = SQRT[\ Vtx_B^2 + Vf_B^2 - 2 \times Vtx_B \times Vf_B \times \cos\ ]}$$

$$\Delta V_B = SQRT[\ 1{,}597^2 + 3{,}075^2 - 2 \times 1{,}597 \times 3{,}075 \times \cos(28)]$$

$$\Delta V_B = 1{,}826 \text{ m/s}$$

From previous equation we know that:

$$\mathbf{\Delta V_T = \Delta V_A + \Delta V_B}$$

$$\Delta V_T = 2{,}455 + 1{,}826$$

$$\mathbf{\Delta V_T = 4{,}281 \text{ m/s}}$$

Three Burn Maneuver

- The first burn is a coplanar maneuver placing the satellite into a transfer orbit with an apogee much higher than the final orbit. When the satellite reaches apogee of the transfer orbit, a combined plane change maneuver is done.
- This places the satellite in a second transfer orbit that is coplanar with the final orbit and has a perigee altitude equal to the altitude of the final orbit.
- Finally, when the satellite reaches perigee of the second transfer orbit, another coplanar maneuver places the satellite into the final orbit. This three-burn maneuver may save propellant, but the propellant savings comes at the expense of the total time required to complete the maneuver.

Correcting Out of Plane Errors

- In some instances, however, a plane change is used to alter an orbit's longitude of ascending node in addition to the inclination.
- An example might be a maneuver to correct out-of-plane errors to make the orbits of two space vehicles coplanar in preparation for a rendezvous.
- If the orbital elements of the initial and final orbits are known, the then the plane change angle is determined by the:

$$\theta = \arccos(a_1 b_1 + a_2 b_2 + a_3 b_3)$$

where,
$$a_1 = \sin i_i \cos\Omega_i$$
$$a_2 = \sin i_i \sin\Omega_i$$
$$a_3 = \cos i_i$$
$$b_1 = \sin i_f \cos\Omega_f$$
$$b_2 = \sin i_f \sin\Omega_f$$
$$b_3 = \cos i_f$$

PROBLEM 5

A spacecraft is in an orbit with an inclination of 30 degrees and the longitude of the ascending node is 75 degrees. Calculate the angle change required to change the inclination to 32 degrees and the longitude of the ascending node to 80 degrees.

SOLUTION,

Given: i_i = 30 degrees

Ωi = 75 degrees

i_f = 32 degrees

Ωf = 80 degrees

- $a1 = \sin(i_i)\cos(\Omega i) = \sin(30)\cos(75) =$ 0.129410
- $a2 = \sin(i_i)\sin(\Omega i) = \sin(30)\sin(75) =$ 0.482963
- $a3 = \cos(i_i) = \cos(30) =$ 0.866025
- $b1 = \sin(i_f)\cos(\Omega f) = \sin(32)\cos(80) =$ 0.0920195
- $b2 = \sin(i_f)\sin(\Omega) = \sin(32)\sin(80) =$ 0.521869
- $b3 = \cos(i_f) = \cos(32) =$ 0.848048

- **$\theta = \arccos(a1 \times b1 + a2 \times b2 + a3 \times b3)$**

$\theta = \arccos(0.129410 \times 0.0920195 + 0.482963 \times 0.521869 + 0.866025 \times 0.848048)$

$\theta =$ 3.259 degrees

Orbit Maneuver Latitude and Longitude

- The plane change maneuver takes place at one of two nodes where the initial and final orbits intersect. The latitude and longitude of these nodes are determined by the vector cross product. The position of one of the two nodes is given by

$$lat_1 = \arctan\left(\frac{c_3}{\sqrt{c_1^2 + c_2^2}}\right)$$

$$long_1 = \arctan\left(\frac{c_2}{c_1}\right) + \begin{bmatrix} 90^\circ \text{ if } c_1 < 0 \\ 270^\circ \text{ if } c_1 > 0 \end{bmatrix}$$

$$\text{where,} \quad \begin{aligned} c_1 &= a_2 b_3 - a_3 b_2 \\ c_2 &= a_3 b_1 - a_1 b_3 \\ c_3 &= a_1 b_2 - a_2 b_1 \end{aligned}$$

- Knowing the position of one node, the second node is simply

$$lat_2 = -lat_1$$

$$long_2 = long_1 \pm 180^\circ$$

PROBLEM 6 Calculate the latitude and longitude of the intersection nodes between the initial and final orbits for the spacecraft in problem 6.

SOLUTION, From problem 5,

$a1 = 0.129410$

$a2 = 0.482963$

$a3 = 0.866025$

$b1 = 0.0920195$

$b2 = 0.521869$

$b3 = 0.848048$

- $c1 = a2 \times b3 - a3 \times b2 = 0.482963 \times 0.848048 - 0.866025 \times 0.521869 = \underline{-0.0423757}$
- $c2 = a3 \times b1 - a1 \times b3 = 0.866025 \times 0.0920195 - 0.129410 \times 0.848048 = \underline{-0.0300543}$
- $c3 = a1 \times b2 - a2 \times b1 = 0.129410 \times 0.521869 - 0.482963 \times 0.0920195 = \underline{0.0230928}$
- $lat_1 = \arctan(c3 / (c1^2 + c2^2)^{1/2})$

$lat_1 = \arctan(0.0230928 / (-0.0423757^2 + -0.0300543^2)^{1/2})$

$lat_1 = \underline{23.965 \text{ degrees}}$

$long_1 = \arctan(c2 / c1) + 90$

$long_1 = \arctan(-0.0300543 / -0.0423757) + 90$

$long_1 = 125.346 \text{ degrees}$

$lat_2 = \underline{-23.965 \text{ degrees}}$

- $long_2 = 125.346 + 180 = \underline{305.346 \text{ degrees}}$

Orbit Rendezvous

- Orbital transfer becomes more complicated when the object is to rendezvous with or intercept another object in space: both the interceptor and the target must arrive at the rendezvous point at the same time. This precision demands a phasing orbit to accomplish the maneuver.
- A *phasing orbit* is any orbit that results in the interceptor achieving the desired geometry relative to the target to initiate a Hohmann transfer. If the initial and final orbits are circular, coplanar, and of different sizes, then the phasing orbit is simply the initial interceptor orbit.
- The interceptor remains in the initial orbit until the relative motion between the interceptor and target results in the desired geometry. At that point, we would inject the interceptor into a Hohmann transfer orbit.

Launch Windows

- Similar to the rendezvous problem is the launch-window problem, or determining the appropriate time to launch from the surface of the Earth into the desired orbital plane. Because the orbital plane is fixed in inertial space, the launch window is the time when the launch site on the surface of the Earth rotates through the orbital plane.
- The time of the launch depends on the launch site's latitude and longitude and the satellite orbit's inclination and longitude of ascending node.

Orbit Maintenance

- Once in their mission orbits, many satellites need no additional orbit adjustment. On the other hand, mission requirements may demand that we maneuver the satellite to correct the orbital elements when perturbing forces have changed them. Two particular cases of note are satellites with repeating ground tracks and geostationary satellites.
- After the mission of a satellite is complete, several options exist, depending on the orbit. We may allow low-altitude orbits to decay and reenter the atmosphere or use a velocity change to speed up the process. We may also boost satellites at all altitudes into benign orbits to reduce the probability of collision with active payloads, especially at synchronous altitudes.

Tsiolkovsky Equation

- From the Newton's equations of motion and momentum:

  Since Thrust is defined as：

$$F = ma = m\frac{dv}{dt} \qquad F = \dot{m} \cdot V_e$$

Thus by equating these two equations:

$$m\frac{dv}{dt} = -\frac{dm}{dt}V_e$$

Tsiolkovsky's Rocket Equation is born:

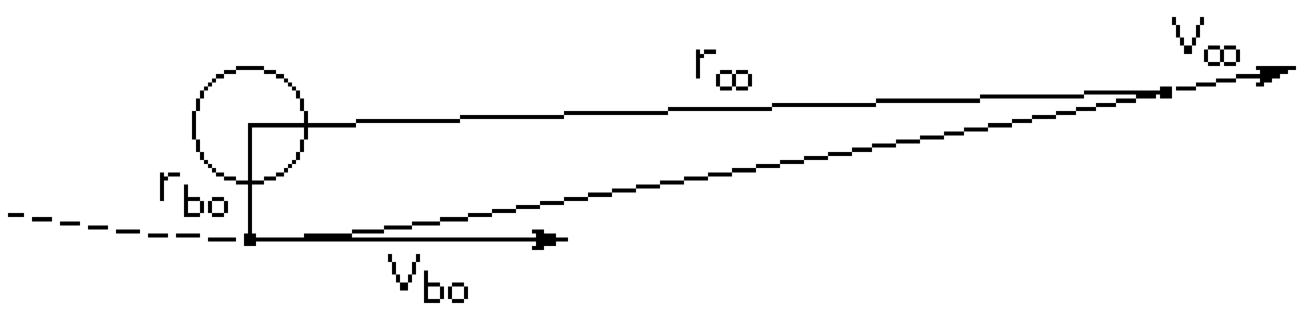
$$\Delta V = V_{exhaust} \ln \frac{M_{initial}}{M_{final}}$$

Delta V Budget

- To an orbit designer, a space mission is a series of different orbits. For example, a satellite might be released in a low-Earth parking orbit, transferred to some mission orbit, go through a series of resphasings or alternate mission orbits, and then move to some final orbit at the end of its useful life. Each of these orbit changes requires energy. T
- The *Delta V budget* is traditionally used to account for this energy. It sums all the velocity changes required throughout the space mission life. In a broad sense the V budget represents the cost for each mission orbit scenario.

Hyperbolic Excess Velocity

- If you give a space vehicle exactly escape velocity, it will just barely escape the gravitational field, which means that its velocity will be approaching zero as its distance from the force center approaches infinity. If, on the other hand, we give our vehicle more than escape velocity at a point near Earth, we would expect the velocity at a great distance from Earth to be approaching some finite constant value. This residual velocity the vehicle would have left over even at infinity is called *hyperbolic excess velocity*.
- We can calculate this velocity from the energy equation written for two points on the hyperbolic escape trajectory – a point near Earth called the *burnout point* and a point an infinite distance from Earth where the velocity will be the hyperbolic excess velocity, v_∞



PROBLEM 7

Calculate the escape velocity of a spacecraft launched from an Earth orbit with an altitude of 200 km.

SOLUTION,

Given:

$r = (6{,}378.14 + 200) \times 1{,}000 = \underline{6{,}578{,}140}$ m

Equation for escaping bodies with hyperbolic orbit was:

$V_{esc} = \text{SQRT}[\ 2 \times GM / r\ ]$

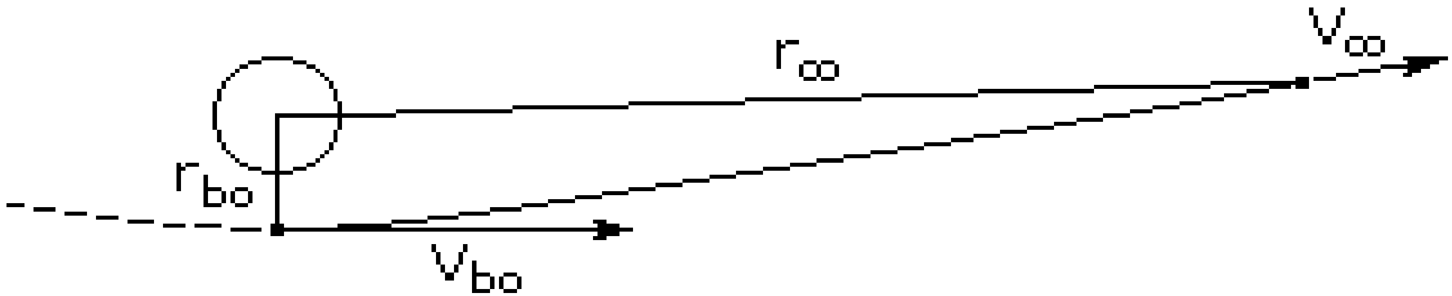
$V_{esc} = \text{SQRT}[\ 2 \times 3.986005 \times 10^{14} / 6{,}578{,}140\ ]$

$V_{esc} = \underline{11{,}009 \text{ m/s}}$

Hyperbolic Excess Velocity Equation

- Solving for v_∞ we obtain:

$$V_\infty^2 = V_{bo}^2 - V_{esc}^2$$



- Note that if v_∞ = 0 (as it is on a parabolic trajectory), the burnout velocity, v_{bo}, becomes simply the escape velocity.

PROBLEM 8

A spacecraft launched from Earth has a burnout velocity of 11,500 m/s at an altitude of 200 km. What is the hyperbolic excess velocity?

SOLUTION, Given:

V_{bo} = 11,500 m/s

From Problem 7,

V_{esc} = 11,009 m/s

Hyberbolic Excess Velocity Equation

$(V_{infinity})^2 = V_{bo}^2 - V_{esc}^2$

$V_{infinity} = SQRT[\ 11{,}500^2 - 11{,}009^2\ ]$

$V_{infinity}$ = 3,325 m/s

Sphere of Influence

- It is a fact, however, that once a space vehicle is a great distance from Earth, for all practical purposes it has escaped. In other words, it has already slowed down to very nearly its hyperbolic excess velocity. It is convenient to define a sphere around every gravitational body and say that when a probe crosses the edge of this *sphere of influence* it has escaped.
- For most purposes, the radius of the sphere of influence for a planet can be calculated as follows:

$$R_p = D_{sp}\left(\frac{M_p}{M_s}\right)^{0.4}$$

- where D_{sp} is the distance between the Sun and the planet, M_p is the mass of the planet, and M_s is the mass of the Sun. Equation is also valid for calculating a moon's sphere of influence, where the moon is substituted for the planet and the planet for the Sun.

PROBLEM 9

Calculate the radius of Earth's sphere of influence.

SOLUTION, From Basics Constants,

D_{sp} = 149,597,870 km

M_P = 5.9737×10^{24} kg

M_S = 1.9891×10^{30} kg

Equation (4.89),

$$R_{Earth} = D_{sp} \times (M_P / M_S)^{0.4}$$

$$R_{Earth} = 149{,}597{,}870 \times (5.9737 \times 10^{24} / 1.9891 \times 10^{30})^{0.4}$$

R_{Earth} = 925,000 km